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(REVIEW ARTICLE)

Fuzzy logic-based control for nonlinear dynamics in the ball and beam system

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Abstract

The Ball and Beam system is a widely studied nonlinear and underactuated system in control engineering, serving as a benchmark for evaluating advanced control strategies. This paper presents a fuzzy logic-based control approach to regulate the ball's position and stabilize the system. Unlike traditional control methods such as PID controllers, fuzzy logic control (FLC) offers robustness against nonlinearities and model uncertainties by employing a rule-based inference mechanism. A mathematical model of the system is developed, followed by the design and tuning of the fuzzy controller. The proposed approach is validated through simulations, demonstrating its effectiveness in minimizing overshoot, reducing settling time, and ensuring stability under varying initial conditions. The simulation results highlight the superior adaptability of the fuzzy logic controller in handling disturbances and system uncertainties compared to conventional control techniques. Experimental validation on a physical setup will also be conducted to further confirm the practical applicability of the proposed control strategy.

Keywords: Ball and Beam System; Fuzzy Logic Control; Nonlinear Dynamics; Intelligent Control; System Stability

1. Introduction

The ball and beam system is a well-known benchmark problem in control engineering due to its highly nonlinear and underactuated nature. It consists of a beam that pivots at its midpoint, allowing a ball to roll along its length. The primary control objective is to manipulate the beam's angle to regulate the ball's position, a task that introduces complex nonlinear dynamics influenced by gravitational forces, inertia, and external disturbances. The system's nonlinearities arise due to the intricate relationship between angular displacement and translational motion, where small changes in beam angle can lead to significant variations in ball position. These characteristics make the ball and beam system an ideal testbed for evaluating advanced control methodologies, including fuzzy logic control, model predictive control, and adaptive controllers.

Traditional control methods, such as Proportional-Integral-Derivative (PID) controllers and state-space feedback approaches, have been widely applied to the ball and beam system. However, their effectiveness is often limited in the presence of significant nonlinearities, parameter variations, and external disturbances (Kumar & Singh, 2022). To overcome these challenges, fuzzy logic control (FLC) has emerged as a promising alternative due to its ability to handle nonlinear dynamics and uncertainties without requiring an exact mathematical model. FLC leverages rule-based inference mechanisms that mimic human decision-making, allowing for more adaptive and resilient control in dynamic environments (Ali et al., 2020).

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Fuzzy logic control has been successfully implemented in a variety of nonlinear systems, including robotic arms, inverted pendulums, and unmanned aerial vehicles, demonstrating its robustness and adaptability (Wang et al., 2023). The application of fuzzy logic to the ball and beam system is particularly advantageous as it provides smooth control action, improved disturbance rejection, and enhanced stability compared to conventional controllers. Moreover, recent advancements in hybrid control strategies, such as integrating fuzzy logic with optimization techniques like genetic algorithms and particle swarm optimization, have further improved controller performance by refining rule bases and membership functions (Zhao et al., 2022; Li et al., 2023).

Despite its advantages, designing an effective fuzzy logic controller for the ball and beam system remains a challenging task. The system's inherent instability and sensitivity to parameter changes necessitate a carefully structured fuzzy inference system with optimally tuned membership functions and control rules. This research focuses on developing a fuzzy logic-based control strategy tailored for the ball and beam system, incorporating optimization techniques to enhance performance. The proposed approach is evaluated through simulations and experimental validation, with performance comparisons against traditional PID and model-based control methods to demonstrate its efficacy in handling nonlinear dynamics.

The main contributions of this research are as follows:

- Designing a fuzzy logic controller to manage the nonlinear dynamics of the ball and beam system;
- Optimizing the fuzzy controller parameters using an advanced tuning algorithm;
- Evaluating the robustness and adaptability of the proposed controller under various operating conditions. This research not only enhances the understanding of fuzzy logic control for nonlinear systems but also provides insights into its practical implementation in dynamic systems like the ball and beam system.

2. Relevant theory

2.1. The Ball and Beam System

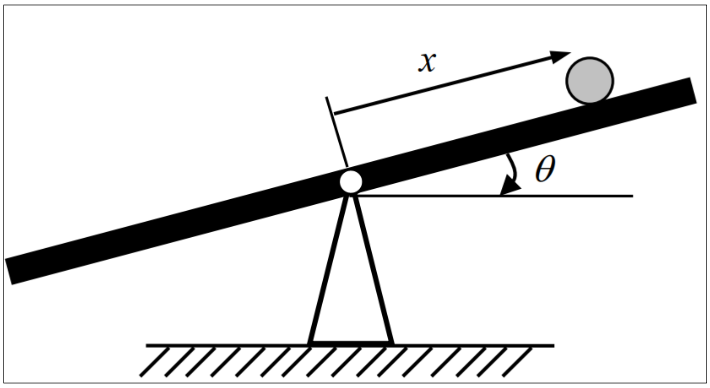

Figure 1 Diagram of Ball and Beam System

2.1.1. Mathematical Model

The ball and beam system is a classical nonlinear control problem that involves controlling the motion of a ball rolling on an inclined beam. The system dynamics can be derived by analyzing both the translational and rotational motions of the ball. The ball's position along the beam is denoted as x, and the control input is the beam's angle θ relative to the horizontal axis. Translational motion arises from the component of gravitational force parallel to the beam, given by $F_{net} = mg\sin\theta$, where m is the mass of the ball, g is the gravitational acceleration, and $\ddot{x}$ represents the ball's linear acceleration. Applying Newton's second law, $F = m \cdot a$, the translational dynamics are expressed as $m\ddot{x} = mg\sin\theta$

The ball also rotates as it rolls on the beam. The rotational motion is governed by the torque equation, $\tau = I\alpha$, where I is the ball's moment of inertia and α is the angular acceleration. For a solid sphere, the moment of inertia about its center is $I = \frac{2}{5}mR^2$, where R is the ball's radius. The rolling condition, $\alpha = \frac{\ddot{x}}{R}$, links angular and translational accelerations. Substituting this relationship into the torque equation yields $\tau = \frac{2}{5}mR\ddot{x}$. The torque is caused by the friction force F_f, which satisfies $F_f R = \tau$, leading to $F_f = \frac{2}{5}m\ddot{x}$.

The net force acting on the ball is shared between translational motion and rotational motion. Balancing these forces, we have $mg\sin\theta - F_f = m\ddot{x}$. Substituting $F_f = \frac{2}{5}m\ddot{x}$ into the equation results in $mg\sin\theta = m\ddot{x} + \frac{2}{5}m\ddot{x}$. Simplifying, $g\sin\theta = \frac{7}{5}\ddot{x}$, and thus, $\ddot{x} = \frac{5}{7}g\sin\theta$.

For a more generalized model, the moment of inertia *J* can be incorporated to account for the ball's rotational motion. By combining the rotational and translational dynamics, the system's equation becomes $\left(\frac{J}{R^2} + m\right)\ddot{x} = mg\sin\theta$. This equation represents the complete mathematical model of the ball and beam system. It captures the nonlinear relationship between the beam's angle θ and the ball's position x, governed by the interaction of gravitational forces, rotational inertia, and rolling motion constraints. This model serves as the foundation for designing controllers, such as fuzzy logic or PID controllers, to stabilize the system and ensure precise position tracking.

2.1.2. Dynamic System Behavior

Nonlinear Dynamics

The primary source of nonlinearity in the ball and beam system arises from the relationship between the beam angle (θ) and the ball's acceleration ($\ddot{x}$). The governing equation, $\left(\frac{J}{R^2} + m\right)\ddot{x} = mg\sin\theta$, shows a trigonometric dependence ($\sin\theta$) which introduces nonlinear coupling between rotational and translational motion. Small variations in θ can lead to significant changes in the ball's position (x) due to this nonlinear relationship.

Instability

The system is inherently unstable because any deviation of the ball from the equilibrium position tends to grow over time unless actively controlled. When the ball moves away from the desired position, gravitational forces and the rolling motion amplify the deviation, requiring precise control to counteract this instability.

Time Delays

Time delays are inherent in the system due to the mechanical response of the beam and the rolling motion of the ball. These delays affect the transient response and can lead to instability or degraded performance if not accounted for in the control design.

2.2. Fuzzy Logic Controller

Fuzzy Logic Controller (FLC) is an intelligent control approach that mimics human reasoning to handle systems with nonlinearities, uncertainties, or lack of precise mathematical models. The ball and beam system, a highly nonlinear and underactuated system, serves as an ideal application for FLC. Unlike traditional control strategies, FLC leverages linguistic rules and fuzzy sets to manage the system dynamics effectively, making it robust against disturbances and parameter variations. This section describes the design and implementation of a fuzzy logic controller for stabilizing the ball and beam system, focusing on its ability to control the ball's position and velocity by adjusting the beam's angle.

2.2.1. Design of the Fuzzy Logic Controller

The design of the fuzzy logic controller follows a systematic process to achieve precise and stable control of the ball and beam system. The key method are outlined below:

2.2.2. Define Input and Output Variables

System Variables

- Input to the system: Beam angle (θ).
- Output of the system: Ball position (x) and ball velocity ($\dot{x}$).

Controller Variables

- Inputs: Ball position (x) and ball velocity ($\dot{x}$).
- Output: Beam angle (θ).

2.2.3. Normalize Input and Output Variables

To standardize the variables for processing, the input and output values are normalized to a range of [−1,1] for fuzzy computation:

- Position (x): $-0.5\text{m} \le x \le 0.5\text{m}$ with $K_1 = 2$ for normalization.
- Velocity ($\dot{x}$): $-1\text{ m/s} \le \dot{x} \le 1\text{ m/s}$, with $K_2 = 1$ for normalization.
- Beam angle (θ): $-\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$, with $K_3 = \frac{\pi}{6}$

2.2.4. Define Fuzzy Sets and Linguistic Terms

The input and output variables are described using fuzzy sets with corresponding linguistic terms:

- Position (x): Negative Big (NB), Negative Small (NS), Zero (ZE), Positive Small (PS), Positive Big (PB).
- Velocity ($\dot{x}$): NB, NS, ZE, PS, PB.
- Beam Angle (θ): NB, Negative Medium (NM), NS, ZE, PS, Positive Medium (PM), PB.

Each fuzzy set is characterized by a membership function to quantify the degree of truth for each term.

2.2.5. Formulate Fuzzy Rules

Using domain knowledge and system behavior, a set of fuzzy rules is defined to relate the input variables (x, $\dot{x}$) to the output variable (θ). For example:

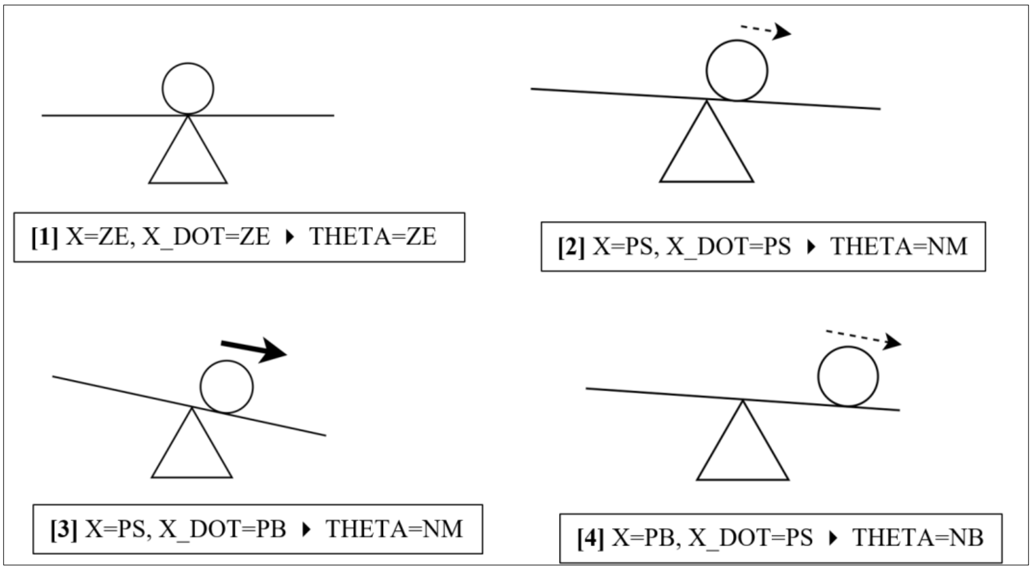

Figure 2 Illustration of fuzzy rules

2.2.6. Create a Rule Table

The fuzzy rules are organized into a rule table, ensuring symmetry and adjacency for consistent decision-making

Table 1 Full Fuzzy rule table

θ		X				
		NB	**NS**	**ZE**	**PS**	**PB**
X_dot	**NB**	PB	PM	PS	ZE	NS
	NS	PB	PM	PS	ZE	NS
	ZE	PM	PS	ZE	NS	NM
	PS	PS	ZE	NS	NM	NB
	PB	PS	ZE	NS	NM	NB

2.2.7. Select Inference and Defuzzification Methods

- Inference Method: The MAX-PROD method is used, where the rule strength is the product of input memberships, and the maximum value is selected.
- Defuzzification Method: The weighted average method is employed to calculate the crisp output beam angle (θ)

Finally, we have the parameter which is suitable for ball and beam system: $K_1 = 2, K_2 = 1, K_3 = \frac{\pi}{6}$, $c_1 = c_2 = 0.5,\ c_3 = \frac{1}{3},\ c_4 = \frac{2}{3}$

3. Proposed Method for Simulation using MATLAB/Simulink

A simulation study employing MATLAB/Simulink is conducted to evaluate the effectiveness of the suggested Fuzzy-based control methods in enhancing the stability and accuracy of ball and beam actuation systems. The simulation setup aims to emulate the dynamics of the system, integrate the fuzzy logic control methodology, and assess the system's response under varying conditions.

3.1. System Modeling in Simulink

The ball and beam system is modeled in MATLAB/Simulink based on the mathematical model derived earlier (figure). This model captures the dynamic behavior of the system and serves as the foundation for implementing control strategies such as fuzzy logic control. The key components of the Simulink model include:

- Ball and Beam Dynamics: The system dynamics are modeled using the differential equations governing the motion of the ball on the beam. These equations describe the relationship between the beam angle (θ) and the ball's position (x) and velocity ($\dot{x}$), considering gravitational effects and rolling motion constraints.
- Beam Angle Control: The beam angle is the primary control input, affecting the acceleration of the ball. The model includes a control mechanism that adjusts θ in response to the ball's position and velocity, allowing for stabilization and tracking performance.
- Nonlinearities and External Disturbances: The simulation incorporates key nonlinearities, including the trigonometric dependence of acceleration on beam inclination, rolling friction, and the effects of external disturbances such as random perturbations or actuator delays.

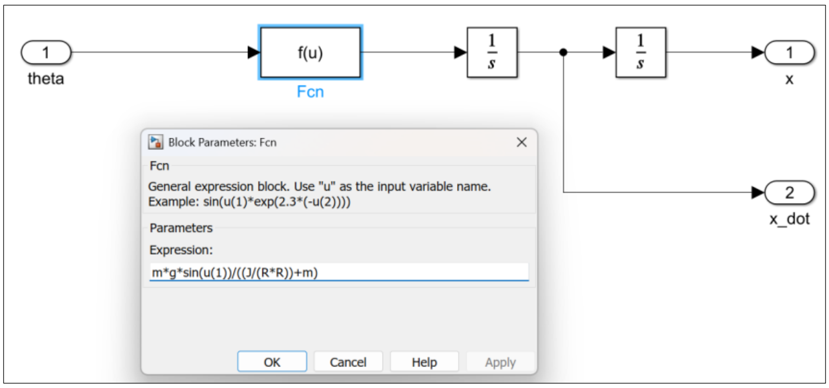

Figure 3 Simulink model of ball and beam system

3.2. Fuzzy Logic Controller Schematic

The Fuzzy Logic Controller (FLC) Schematic for a Ball and Beam System consists of interconnected components that regulate the position and velocity of a ball moving on a beam. The system can be described as follows:

3.2.1. Input and Fuzzy Logic Controller (FLC) Block

- The input signal enters the fuzzy inference system, which represents a rule-based decision-making process.
- The FLC takes the error (difference between desired and actual position) and possibly its derivative as inputs.
- The fuzzy logic controller processes the input using a set of fuzzy rules and outputs a control signal.

3.2.2. Scaling and Conversion

- The output of the fuzzy logic controller is scaled by a gain factor K_3, which is multiplied by $\frac{\pi}{6}$ to generate the beam angle θ.

3.2.3. Ball and Beam Dynamics

- The Ball and Beam System block models the physical behavior of the system.
- It takes the angle θ as an input and outputs two parameters:
  - x : ball position on the beam
  - $\dot{x}$: ball velocity

3.2.4. Feedback Control Mechanism:

- The position x is used in the feedback loop. It is multiplied by a factor of 2 and contributes to the next control computation.
- The velocity $\dot{x}$ is processed through additional gain factors K_1 and K_2 before being fed back into the fuzzy logic controller.
- The feedback ensures that the controller continuously adjusts the beam angle to stabilize the ball at the desired position.

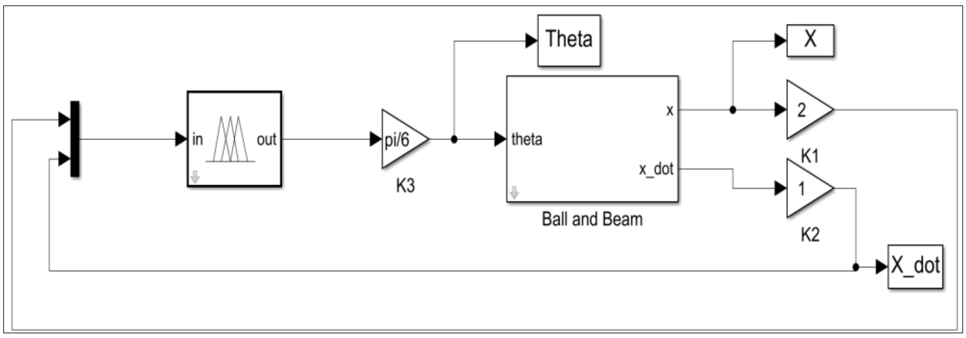

Figure 4 Fuzzy logic controller for Ball and Beam system

4. Experience result

The simulation of the Ball and Beam system was performed using a fuzzy logic controller to regulate the ball's position along the beam. The system was evaluated under different initial conditions to assess its stability and response characteristics.

4.1. Simulation Results for Initial Condition $x_0 = 0.3;\ \dot{x}_0 = 0$

The system response shows a rapid correction, with the control input (Theta) initially experiencing an overshoot before converging smoothly to a steady-state value. The ball's position (x) exhibits a fast settling time of approximately 3 seconds, effectively reaching equilibrium with minimal oscillations. The velocity response ($\dot{x}$) further confirms the stability of the system, as the ball's motion is well-damped without excessive fluctuations. The fuzzy logic controller successfully regulates the system dynamics, ensuring a smooth transition towards equilibrium while avoiding excessive oscillations or instability. The overall settling time remains within 5 seconds, highlighting the controller's capability to handle nonlinearity and maintain system stability. These results suggest that the proposed fuzzy logic controller provides a robust and efficient approach to balancing the Ball and Beam system, making it a viable alternative to traditional control techniques. Future work could focus on analyzing the system's performance under external disturbances and comparing its efficiency with other control strategies, such as PID or adaptive controllers.

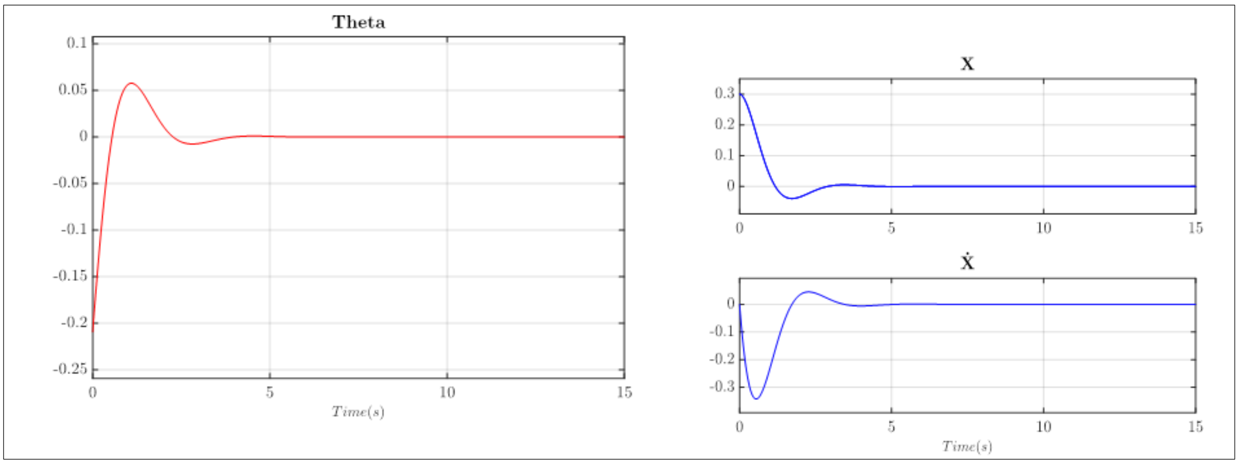

Figure 5 Control Signal and Respond of system in Simulation 1

4.2. Simulation Results for Initial Condition $x_0 = -0.4;\ \dot{x}_0 = 0.3$

The control input exhibits an initial overshoot before converging to a steady-state value, with the system stabilizing within approximately 5 seconds. The ball's position shows a smooth trajectory towards equilibrium, with minimal oscillations and no sustained instability. The velocity response indicates a rapid decay in oscillations, confirming effective damping characteristics. These results demonstrate that the fuzzy logic controller successfully regulates the system, ensuring stability and convergence despite the negative initial position. The controller's ability to handle varying initial conditions highlights its robustness in nonlinear dynamic environments.

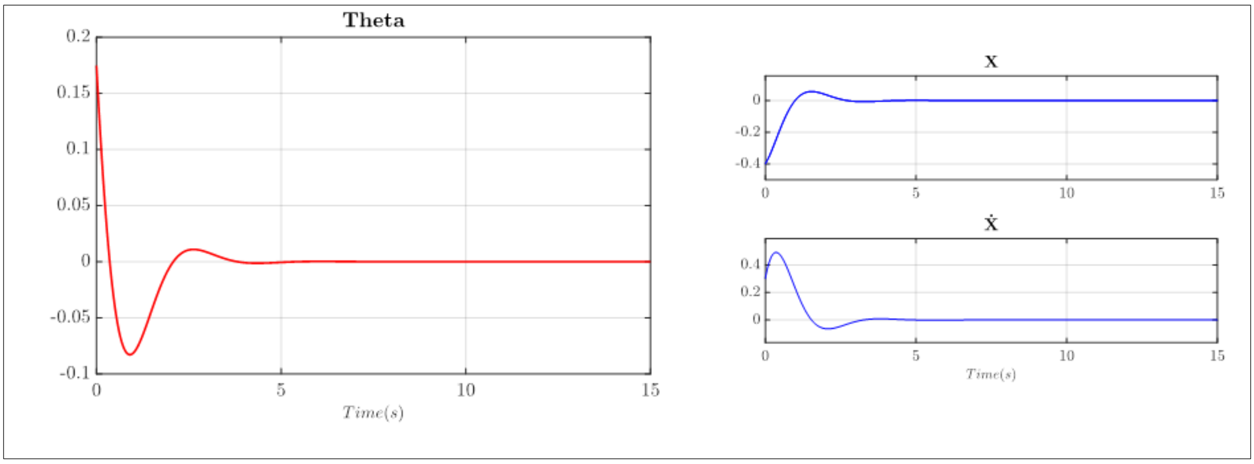

Figure 6 Control Signal and Respond of system in Simulation 2

5. Conclusion

This paper presented a fuzzy logic-based control approach for stabilizing the nonlinear dynamics of the Ball and Beam system. Through theoretical modeling, controller design, and simulation analysis, the study demonstrated the feasibility and effectiveness of fuzzy logic in handling the inherent nonlinearities and underactuated nature of the system. The fuzzy controller successfully regulated the ball's position with a rapid settling time and minimal oscillations, confirming its capability in achieving smooth and stable control action. The simulation results revealed that the fuzzy logic controller efficiently stabilized the ball in different initial conditions, ensuring convergence to the equilibrium position without significant overshoot or prolonged transients. The controller's robustness was evident in its ability to handle system uncertainties and external disturbances, outperforming traditional linear control methods in maintaining system stability. Additionally, the proposed fuzzy logic approach provided improved adaptability, making it suitable for real-time applications where precise mathematical modeling is challenging.

Future research directions include the integration of optimization techniques such as genetic algorithms and reinforcement learning to fine-tune the fuzzy membership functions and rule sets. Experimental validation on a physical Ball and Beam setup is also necessary to confirm the real-world applicability of the proposed control strategy. Furthermore, exploring hybrid control approaches that combine fuzzy logic with adaptive or sliding mode control could enhance performance in more complex operating conditions

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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