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(Research Article)

Select an equation to calculate the dead time for a radiation counter system

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Abstract

Dead time is one of three critical parameters in the radiation measurement system that employs a GM counter tube. Dead time is determined using both accurate and approximation approaches. However, calculating the uncertainty of dead time using the exact method is challenging when instructing students. In this work, we present a method for selecting an equation from the approximation method to compute the dead time and its uncertainty, ensuring deviations are less than 5% when compared to the exact solution method.

Keywords: Dead time; Radiation counter; Select an equation; Exact solution; Approximate solution

1. Introduction

In a radiation counter system, the minimal separation time of two events before being recorded as independent is referred to as the dead time of the counting system. This dead time is determined by the detector's attributes and the behavior of the pulse processing circuitry, which reduces counting errors [1]. There are various ways to calculate the dead time of a radiation counter system, including the two-source method, the pulser method, the decaying source method, and the increasing power method [2]. As indicated by the authors, the two-source method with a non-paralyzing model has been more widely adopted to study the dead time of radiation-detection systems [2]. In this situation, the dead time represents the quadratic equation's root [2-4]. An equation expressing the exact solution to this quadratic problem is rather complex to calculate the dead time and its uncertainty. In published reports and lectures, around four equations defining the approximate solution are utilized to calculate the counting system's dead time [2, 4] with no uncertainty [2, 5-9].

The purpose of this study is to offer a method for selecting an approximate equation to compute the dead time and its uncertainty for a radiation measurement system that ensures equation simplicity and a result deviation of less than 5% when compared to the exact solution equation.

2. Material and methods

2.1. Methods

The dead time of the radiation counter system, according to the two-source method with a non-paralyzing model, is described by the following formula [2-4]:

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$$\frac{x}{1-x\tau}+\frac{y}{1-y\tau}=\frac{z}{1-z\tau}+\frac{b}{1-b\tau} \quad \text{..............(1)}$$

In this context, x, y, z, and b represent the measured count rates for source 1, source 2, the combined sources 1 and 2, and the background, respectively; τ denotes the counter system's dead time.

The dead time is obtained from equation (1), with the exact solution [2-4] shown in Equation (2):

$$\tau=\frac{1}{2A}\left(-B-\sqrt{B^2-4AC}\right) \quad \text{.................(2)}$$

A = xy(z + b) – zb(x + y)

B = 2(zb – xy)

C = x + y – z – b

Although the dead time can be determined precisely using Equation 2, calculating its value using Equation 2 is not easy, especially the equation for determining its uncertainty. As a result, several authors used the approximate solution method for Equation 1 to derive the dead time calculation Equations (3a), for Refs. [2, 4, 5, 10-15], (4a) for Refs. [2, 16-20], (5a) for Refs. [2, 4], and (6a) for Refs. [2, 10, 21], respectively. According to Ref. [22], the total uncertainty is the quadratic sum of partial errors; hence, the uncertainty of the dead time in equations (from 3(a) to 6(a)) can be stated in terms of Equations (3b), (4b), (5b), and (6b):

$$\tau=\frac{x+y-z-b}{2(x-b)(y-b)} \quad \text{.................(3a)}$$

$$\sigma_\tau=\frac{1}{2(x-b)^2(y-b)^2}\times \sqrt{\begin{aligned}&(y-b)^2(y-z)^2\sigma_x^2+(x-b)^2(x-z)^2\sigma_y^2+(x-b)^2(y-b)^2\sigma_z^2\\&+\left((x-b)(y-b)+(x+b-z-b)(2b-x-y)\right)^2\sigma_b^2\end{aligned}} \quad \text{..............(3b)}$$

$$\tau=\frac{x+y-z-b}{z^2-x^2-y^2} \quad \text{........................ (4a)}$$

$$\sigma_\tau=\frac{1}{\left(x^2+y^2-z^2\right)^2}\times \sqrt{\begin{aligned}&\left(x^2+y^2-z^2-2x(x+y-z-b)\right)^2\sigma_x^2+\left(x^2+y^2-z^2-2y(x+y-z-b)\right)^2\sigma_y^2\\&+\left(x^2+y^2-z^2-2z(x+y-z-b)\right)^2\sigma_z^2+\left(x^2+y^2-z^2\right)^2\sigma_b^2\end{aligned}} \quad \text{(4b) ....}$$

$$\tau=\frac{2(x+y-z)}{(x+y)z} \quad \text{................. (5a)}$$

$$\sigma_\tau = 2\sqrt{\frac{\sigma_x^2}{(x+y)^4} + \frac{\sigma_y^2}{(x+y)^4} + \frac{\sigma_z^2}{z^4}}$$ …………………(5b)

$$\tau = \frac{1 - \sqrt{1 - \frac{z(x+y-z)}{xy}}}{z}$$ …………………..(6a)

$$\sigma_\tau = \frac{1}{2xyz^2(xy - z(x+y-z))}\sqrt{D+E+F}$$ …………………..(6b)

$$D = \frac{yz^2}{x}(xy - z(x+y-z))(-xy + x(y-z) + z(x+y-z))\sigma_x^2$$

$$E = \frac{xz^2}{y}(xy - z(x+y-z))(-xy + y(x-z) + z(x+y-z))\sigma_y^2$$

$$F = x^2y^2\left(\begin{array}{l} -z(x+y-2z)\sqrt{\frac{xy - z(x+y-z)}{xy}} \\ +2(xy - z(x+y-z))\left(1 - \sqrt{\frac{xy - z(x+y-z)}{xy}}\right)\end{array}\right)^2 \sigma_z^2$$

where σ_x, σ_y, σ_z, and σ_b are the errors of measured count rates of source 1, source 1, the combined sources 1 and 2, and the background, respectively.

2.2. Materials

To select an equation for calculating the dead time of a radiation counter system, we gathered data from 17 books and 5 articles, which are listed in the reference section. According to these papers, there are five standard equations for computing dead time: one for estimating the exact value and four for approximating the values. In addition, we collected ten data sets for dead time calculation and used them to evaluate and choose the optimum approximation equation for the measurement system's dead time.

3. Results and discussion

Table 1 displays the count rates of the radiation sources according to the two-source method taken from Refs. [1, 2, 5-10, 21]. We found that this data may be separated into three groups based on the unit and result of the measurement. It consists of the cpm (count per minute) unit without errors, the cps (count per second) unit with errors, and the cps unit without errors.

Table 1 Count rates according to source method are taken from the references

No.	Ref.	Count rate				Deviation*, [%]
		Source 1	Source 2	Source 1 and 2	Background	
[cpm], without the error						
1	[5]	9360	8220	16860	60	12
2	[6]	15000	14000	26000	50	7
3	[2]	106800	93600	173400	30600**	12

[cps], with the error						
4	[21]	209.72 ± 0.05	126.80 ± 0.15	329.13 ± 0.67	0.03 ± 0.01	40
5	[1]	264.0778 ± 0.27084	257.3617 ± 0.26737	473.7786 ± 0.36277	0.0922 ± 0.00506	3
6	[10]	395 ± 3	334 ± 3	655 ± 3	-	15
[cps], without the error						
7	[7]	124	78	197	-	37
8	[8]	126.94	121.68	243.15	1.306	4
9	[9]	1323	936	2028	-	29
10	[9]	3511	2951	4794	-	16

Note: Ref. is the reference, * is the deviation of the count rates between the sources, ** is counted in 30 minutes.

The values of dead time are best estimated by Equation 2 and approximated by Equations 3-6. The results of these estimates are shown in Table 2.

Table 2 Estimated results of the dead time

No.	**Ref. , Dead time τ [μs]**	**This work, Dead time τ [μs]**				
		Exact estimation	**Approximate estimation, using equations**			
		Equation 2	**3a and 3b**	**4a and 4b**	**5a and 5b**	**6a and 6b**
[cpm], without the error						
1	[5], 241	271.13	260.91 ± 70.53	306.78 ± 97.50	291.50 ± 75.08	292.79 ± 75.43
2	[6], -	472.51	424.38 ± 30.31	694.18 ± 81.02	477.46 ± 37.56	478.10 ± 37.61
3	[2], 91.51	91.51	79.59 ± 1.63	157.45 ± 6.36	93.24 ± 2.14	93.72 ± 2.15
[cps], with the error						
4	[21], 142.28 ± 13.53	136.35	133.30 ± 12.96	146.27 ± 15.60	131.44 ± 12.68	142.28 ± 13.53
5	[1], 385.4 ± 1.80	385.37	350.21 ± 3.52	537.54 ± 8.28	385.85 ± 4.27	385.92 ± 4.28
6	[10] 282 ± 20	312.42	280.45 ± 17.12	458.36 ± 45.98	309.95 ± 21.23	312.42 ± 21.40
[cps], without the error						
7	[7], 260	265.42	258.48 ± 1007.35	288.20 ± 1252.32	251.29 ± 1004.48	265.42 ± 1062.55
8	[8], 140	140.01	137.67 ± 721.56	147.48 ± 829.89	180.97 ± 733.81	181.05 ± 734.15

9	[9], 1323	104.30	93.27 ± 23.75	155.41 ± 65.86	100.85 ± 28.75	104.30 ± 29.88
10	[9], 3511	108.94	80.49 ± 3.84	856.74 ± 426.07	107.69 ± 7.15	108.94 ± 7.29

As seen in Table 2, the estimated dead time by Equation 4a differs the most from the predicted dead time by Equation 2. Therefore, we should not use Equation 4a to calculate the dead time for the radiation measurement system. The results in Table 2 reveal that the dead times estimated by Equations 5a and 6a are roughly equal. However, Equation 6a is more complex than Equation 5a. Therefore, to estimate the dead time, we should use Equation 5a rather than 6a.

At this point, we need to consider whether to use Equation 3a or 5a to estimate the dead time. We also find that the choice of Equation 3a or 5a depends on the product of the dead time and the recorded count rates from the radiation source. It is easy to see that Equation 5a is the simplest equation to use for estimating the dead time. The results of the product of the dead time (calculated by Equation 5a in Table 2) and the count rates (shown in Table 1) are listed in Table 3.

Table 3 Relationship between dead time and measured count rates

No.	Ref.	Dead time according to equation 5, τ [μs]	xτ	yτ	zτ
1	[5]	291.50	0.05	0.04	0.08
2	[6]	477.46	0.12	0.11	0.21
3	[2]	93.24	0.17	0.15	0.27
4	[21]	131.44	0.03	0.02	0.04
5	[1]	385.85	0.10	0.10	0.18
6	[10]	309.95	0.12	0.10	0.20
7	[7]	251.29	0.03	0.02	0.05
8	[8]	180.97	0.02	0.02	0.04
9	[9]	100.85	0.13	0.09	0.20
10	[9]	107.69	0.38	0.32	0.52

Based on the data in Table 3, our analysis suggests that Equation 5a should be used when the product of dead time and count rate of the source with a lower value ($y\tau$) is higher than or equal to 0.9, and Equation 3a when the value is smaller than or equal to 0.4. Consequently, we have not collected data on the value of $y\tau$, which ranges from 0.41 to 0.89. To enhance the study presented in this article, we will conduct further investigations on the value of $y\tau$ in the range just mentioned above.

Tables 1 and 3 show that the approximation equation used to compute the dead time for the counting system is unaffected by the difference in count rates between the two sources. Equation 3a is utilized when the counting rate difference between the two sources is between 4 and 40%, and Equation 5a when that difference is between 3 and 29%. By using the method described above, the deviation in dead time determined by Equation 3a or Equation 5a compared to Equation 2 is less than 4%.

4. Conclusion

In this work, we looked at how to choose an equation from the approximation solution approach to determine dead time and its uncertainty for a radiation counter system that used a GM tube. The results of our analysis reveal that the choice of the equation to calculate dead time does not depend on the deviation of the count rates between the two sources. Still, it depends on the product of the lower count rate (when compared to the remaining source) and estimated dead time (using Equation 5a). These findings are useful for teaching students in the field of nuclear radiation detection, as they assist lecturers and students in avoiding situations where the data used to calculate the dead time and its uncertainty of the measurement system can have uncertainties of up to 400% (data taken from Refs. 7 and 8), as shown in Table 2.

To choose an equation for determining the dead time of a radiation detection system, we first compute the dead time using equation 5a, then multiply the result by the count rate of one of the two sources with the lowest value. If this product is larger than or equal to 0.9, the dead time and its uncertainty should be calculated according to Equations 5a and 5b, respectively; otherwise, if the calculated product is less than or equal to 0.4, the dead time and uncertainty should be computed using Equations 3a and 3b, respectively.

Compliance with ethical standards

Disclosure of conflict of interest

The author has declared that no competing interest exists

Statement of ethical approval

This study does not contain any studies with human or animal subjects performed by any of the authors.

Author Contributions

The first draft of the manuscript was written by Pham Dang Quyet, author read and approved the final manuscript.

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