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(Research Article)

Game theory model analysis for solid waste optimization in Enugu Municipal, Enugu State Nigeria

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Abstract

The use of Game theory model to optimize the planning and management of Municipal Solid Waste (MSW) in Enugu is intended to solve the problem of maximizing the solid waste generated for the economic benefit of the state and her inhabitants. This would ensure proper solid waste handling, recovery, reduction, re-use and recycling to wealth. The aim is to develop optimization solution strategies for solid waste management using Game theory decision model theory. The existing solid waste volume and characteristics were used at designated dump sites in Enugu to estimate the solid waste generated per day and game theory optimization model used to create wealth for Enugu State. The methodology used include characterization of solid waste deposited at selected dumpsites and estimation of the waste volume generated per capita per day projected for fifty (50) years based on 2006 census figure for Enugu municipal which comprise Enugu North, Enugu East and Enugu South Local government areas of Enugu State. The estimated population projection based on record from Enugu State Waste Management Authority (ESWAMA) shows that Enugu municipal would generate over 3234tons per day by March 2025. The summary of costs/benefits shows that minimax is 2.16 while the maximum is 5.19 without a saddle point so the linear programming of game theory was used to calculate the value of the game. The result shows that the financial benefit under the worst condition will be N18.284 trillion per annum.

Keywords: Game Theory; Optimization; Municipal Solid Waste; Management; Planning

1. Introduction

The management of solid wastes and associated wastes are essential to maintain healthy and sustainable environment. These are crucial for sustainable economic growth. Although solid waste constitutes nuisance to the environment but proper planning and management can create wealth for the community where they are generated.

In some cities of developing countries bulk of solid wastes are seen on streets and in open spaces. This waste to say the least disfigures the city, creates an eyesore and also poses tremendous health hazards to the public. The under estimation of the amount of solid wastes generated is one basic problem that has hampered most planning and management of solid wastes in most cities. This has resulted to poor design calculation which led to incorrect capacity of waste management systems. The possession of corrected and adequate information on the rate of generation and composition of wastes generated will make it easy to propose and implement an effective method of management (Aramabi, 1998). Therefore, the generation rate and composition of the wastes generated in Enugu must be first identified in order to know the best management options to use. Urban solid waste planning and management is one of the most serious problem faced by urban centres all over the world. The management of the quantity of Municipal Solid Waste (MSW), which is an indicator of an urban lifestyle, has been a serious issue of concern all over the Enugu

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community as a result of great increases in population, rise in the living standard, etc. These have increased Solid waste generation rate in Enugu and made solid waste composition more complex and heterogeneous. This problem is worse in public places such as Schools, Markets, Mechanics sites etc. in which Municipal Solid Waste Management is highly neglected.

The projection from the 2006 census figure of 722664 (NPC Report, 2006) shows that the population of Enugu municipal would be at least 900319 in 2020 and 973,844 in March 2025. Enugu State population was 3,267,837 in 2006 with a population density of 268 persons per square kilometers while the average national density is about 96 persons per Sq.km. The density for the urban concentration range from 300-600 persons per Sq.km (NPC, 2006). Enugu Master Plan (1992) stated that high density neighbourhoods contain about seventy percent (70%), medium density areas contains about twenty eight percent (28%) while low density contain about two percent (2%) of the urban population.

Tremendous increase in volume and types of solid waste as a result of continuous economic growth and urbanization is becoming a growing problem for national and local governments (Carwyn, 2003). Solid waste survey and characterization are special tools in bringing to light the generation rate and composition of solid waste (Sincero and Sincere, 2006). It is too common to have solid waste disposed in Enugu without attempting to explore the wealth creation options of solid waste management. Management methods such as recovery, recycling, and reuse are very important tools for creating wealth from waste. All these solid waste management alternatives cannot be effective without the correct knowledge of solid waste compositions in Enugu. Oyinlola(1999) stated that it is not essentially every composition of solid waste that can be further utilized as resources for wealth creation. Therefore, solid waste components in Enugu must be well classified to make the compositions readily differentiable to intended stakeholders.

Tchobanoglous and Kreith (2002) stated that it is essential for adequate attention to be paid to all components of solid waste management so as to ensure cost-effectiveness and sustainability of the management system. There are ten components of effective solid waste management which include:

(i) Waste Generation, (ii) Waste storage, (iii) Waste collection, (iv) Waste reduction, (v) Waste transfer, (vi) Waste recycling, (vii) Waste re-use, (viii) Waste resources recovery(ix) Waste treatment/processing and (x) Waste disposal.

The Four (4) of Solid Waste Operations include:

- **Solid Waste Reduction** which can be described as any change in the design, manufacture, purchase, or use of materials or products (including packaging) to reduce their volume and amount of toxicity before they become municipal solid waste. Source reduction also refers to the reuse or recycling of products or materials which does not exist.
- **Solid waste reuse** which is expressed as using a waste product without further transformation and without changing its shape or original nature. Different types of solid wastes can be reused, such as bottles, old clothes, books and anything else that is used again for a similar purpose to that originally intended. These practices are done by scavengers without proper coordination by the Waste Management Authority.
- **Solid waste recycling** which is explained as the material that is reprocessed before being used to make new products. Recycling means treating the materials as valuable resources rather than as waste. These facilities are neither provided by individual business concerns or the public authorities at any level.
- **Solid waste recovering** is referred to as the collection and reuse of disposed materials such as empty beverage containers. The materials from which the items are made can be reprocessed into new products. Therefore, solid waste recovery is concerned with the range of garbage materials arising from animal and human activities that are discarded as unwanted and useless. Solid waste is generated from industrial, residential and commercial activities in a given area and may be classified in a variety of ways. For example, landfills are typically classified as sanitary, municipal, construction and demolition, or industrial waste sites.
- **Waste can be categorized** also based on material, such as plastic, paper, glass, metal and organic wastes. Waste categorization may also be based on hazard potential, including radioactive, flammable, infectious, toxic or non-toxic wastes. Categories may also pertain to the origin of the waste, whether industrial, domestic, commercial or construction and demolition. Regardless of the origin, content or hazard potential solid waste must be managed systematically to ensure environmental best practices. As solid waste management is a critical aspect of environmental hygiene, it must be incorporated into environmental planning.

1.1. Issues in Solid Waste Management

The Management of Solid Wastes involves the following major issues which include: (1) increasing waste quantities; (2) wastes not reported in the national MSW totals; (3) lack of clear definitions for solid waste management terms and functions; (4) lack of quality data, (5) need for clear roles and leadership in federal, state, and local government; (6) need for even and predictable enforcement regulations and standards, and (7) resolution of intra-country, interstate, and inter-country waste issues for MSW and its components. Figure 1 shows the possible sources of solid waste in a community.

It is assumed that, the term municipal solid waste (MSW) normally include all of the wastes generated in a community, with the exception of waste generated by municipal services, treatment plants, and industrial and agricultural processes.

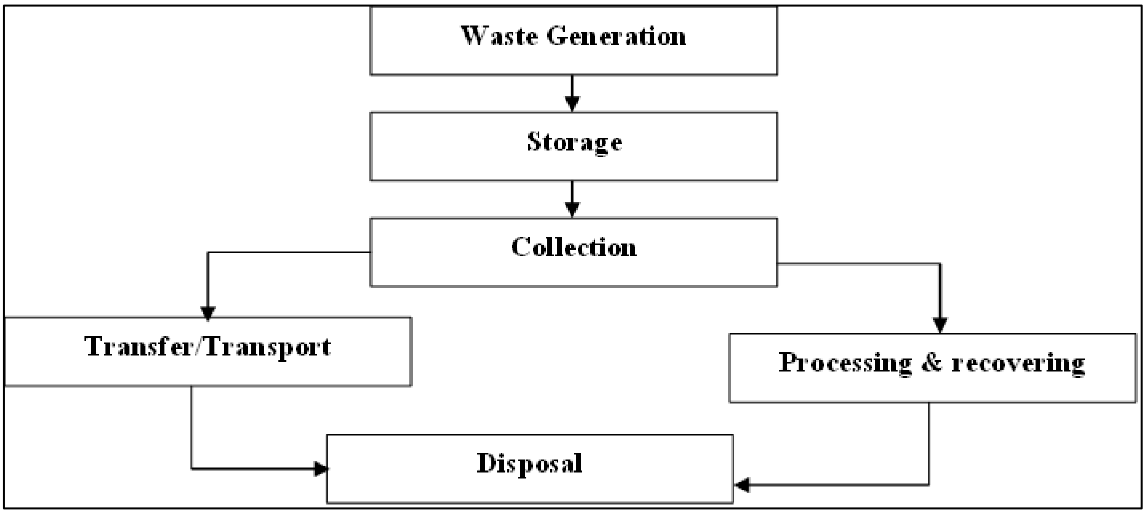

Figure 1 Interrelationship of Functional Elements of a Solid Waste Management System

1.2. Aim and Objectives of the Study

The aim of the is to use Game theory model analysis to optimize solution strategies for solid waste management in Enugu municipal. The objectives are to quantify solid wastes volume, determine the solid wastes characteristics and apply game theory model to optimize the most cost effective solid wastes management system in order to create wealth for the state.

2. Literature Review

The literature review is based on the concept of Game theory model which was used for optimization of solid waste generation in Enugu municipal.

2.1. Games Theory Model

Game is referred to as a situation of conflict and competition in which two or more competitors (or participants) are involved in decision making in anticipation of certain outcomes over a period of time. In game, competitors referred as players may be an individual or a group of individuals, or an organization. When using theory of games to select an optimal strategy of two or more competitors in a competitive and conflicting decision environment, it can be used in pricing of products, various television networks, success of a business tax strategy and success of an advertising/marketing campaign etc.

The theory of games as an area of academic study provides a series of mathematical models that may be useful in explaining interactive decision-making concepts where two or more competitors are involved under conditions of conflicts and competition. Although, it is limited in scope as a practical tool, the models provide an opportunity to a competitor to evaluate not only his personal alternatives (courses of action) the evaluation of the opponent's (or competitor's) possible choices in order to win the game is also considered (Hillier and Lieberman, 2020).

2.1.1. Factors dependent upon classification models in the theory of games

- **Number of players:** When two players (competitors) are involved, it is referred to as a two-person game otherwise n-person game for more players.
- **Zero-sum game:** Means that the sum of gains to one player is exactly equal to the sum of losses to another player such that the sum of gains and losses equals zero, otherwise it is called a **non-zero-sum game**.

- **Strategy**: is the list of all possible actions (move or courses of action) that the player will take for every payoff (outcome) that might arise.
- **Optimal strategy:** is the particular strategy by which a player optimizes his gains or losses without knowing the competitor's strategies. If the maximum value equals the minimal values, the game is said to have a saddle (equilibrium) point and the corresponding strategies are called optimal strategies.
- **Value of the game:** is the expected outcome per play when the players follow their optimal strategy.
- **Pure strategy:** is the decision rule which is always used by the player to select the particular strategy (course of action). Each player knows in advance all strategies out of which he always selects only one particular strategy regardless of the other player's strategy whose objective is either to maximize gains or minimize losses.
- **Mixed strategy:** implies that the courses of action are selected on a particular occasion with some fixed probability. There is a probabilistic situation with the objective of the players to maximize expected gains or to minimize expected losses by making choice among pure strategies with fixed probabilities.

A mixed strategy for a player with two or more possible courses of action is the set S of n non-negative real numbers **(**probabilities**)** whose sum is unity, n being the number of pure strategies of the player. If Pj (j = 1, 2, ..., n) is the probability with which the pure strategy, j would be selected, then, $S = (P_1, P_2, ..., P_n)$ where $P_1 + P_2 + ...+ P_n = 1$ and $Pj \geq 0$ for all j.

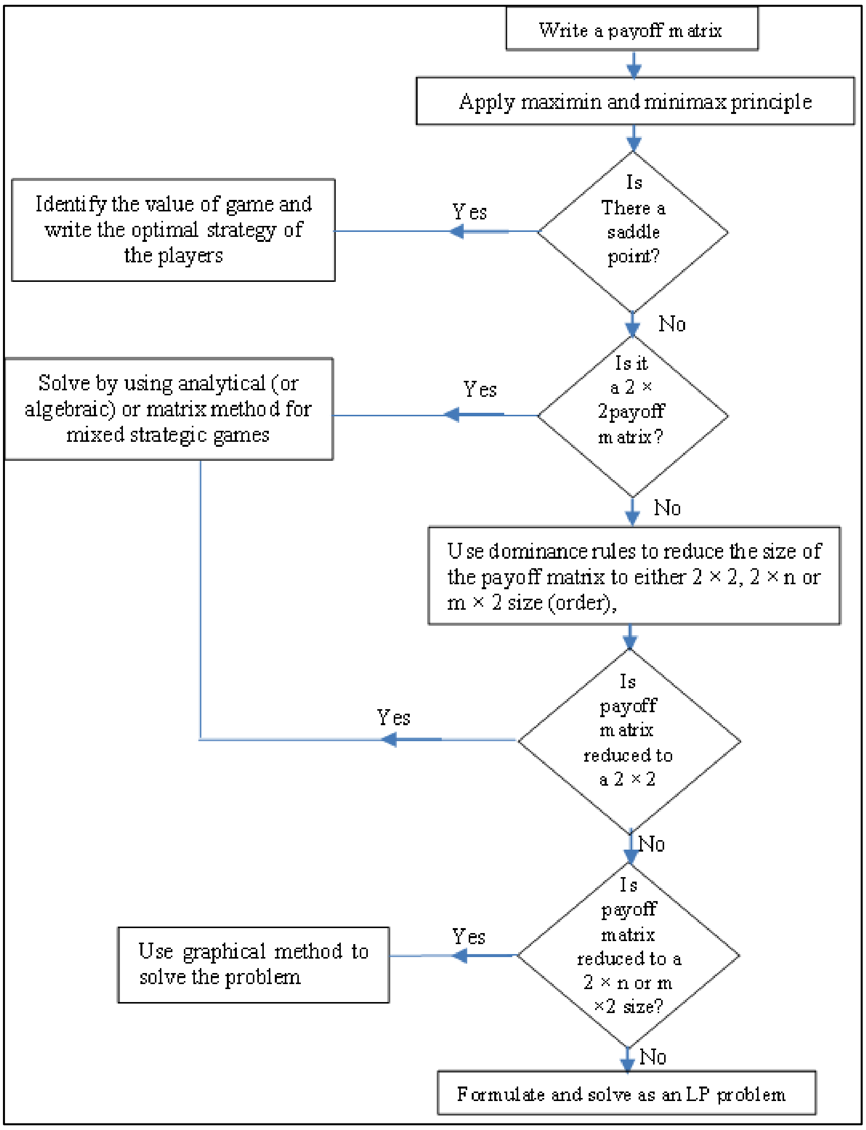

Figure 2 Flow chart of Game Theory approach

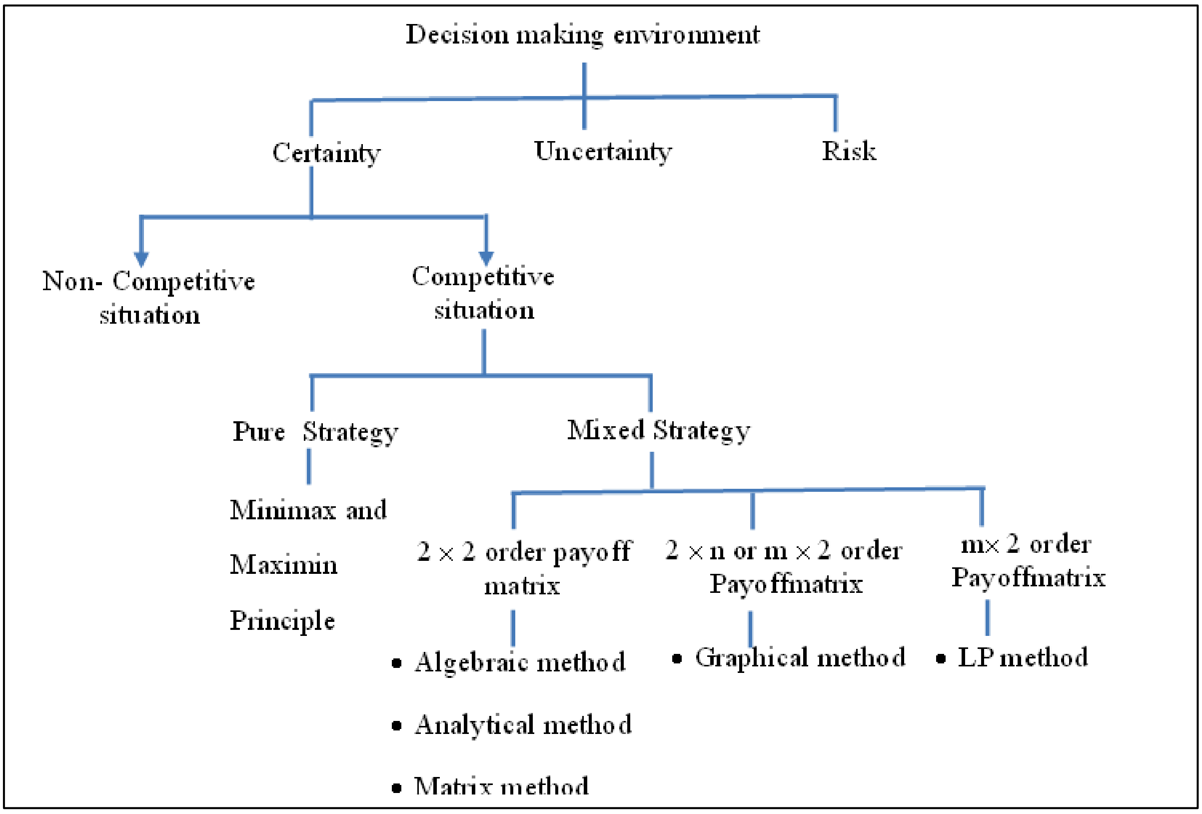

Figure 3 Various methods to find value of game under decision making environment of certainty.

- **Two-person zero-sum games** is a game, with only two players, say player A and player B, if one player's gain **is** equal to the loss of other player such that the total sum is zero.
- **Pay offs** represents a quantitative measure of satisfaction which a player gets at the end of the play.
- **Pay off matrix**: is the Pay offs in terms of gains or losses when players select their particular strategies which are represented in the form of a matrix.
- **Value of the game** is referred to as the expected payoff at the end of the game when each player uses his optimum strategy, i.e. the amount of payoff V at an equilibrium point. The value of the game in general satisfies the equation, maximum value ≤ V ≤ minimum value.
- **Saddle point** occurs in a game when the minimum of the column maxima and the maximum of the row minima are equal. A game may have more than one saddle points while a game with no saddle point is solved by choosing strategies with fixed probabilities.
- **A fair game** is when in a game the lower (maximin) and upper (minimax) values are equal and both equals zero.
- **A strictly determinable game** is when the lower (maximin) and upper (minimax) values of the game are equal and both equal the value of the game.
- **Maximin Principle** means that for player A minimum value in each row represents the least gain (payoff) to him if he chooses his particular strategy known as the row minima. He selects the strategy the largest among the row minimum values. The choice of player A is referred to as the **Maximin Principle** and the corresponding gain is called the **maximin value of the game**.
- **Minimax Principle** means that for player B who is assumed to be the looser, the maximum value in each column represents the maximum loss to him if he chooses his particular strategy. It is referred to as column maxima in the payoff matrix. He now selects the strategy that gives minimum loss among the column maximum values. This choice of player B is the **minimax principle**, and the corresponding loss is the minimax value of the game.
- **The Rules of Dominance** is the strategy used to reduce the size of the payoff matrix. These rules help in deleting certain rows and/or columns of the payoff matrix which are inferior (less attractive) to at least one of the remaining rows and columns (strategies) in terms of payoffs to both the players.

2.1.2. Linear programming method of Game theory model

There is some relationship between Game theory and linear programming. Two-person zero-sum games can also be solved by linear programming technique. It has an additional advantage of being able to solve mixed strategy games of larger dimension payroll matrix. To illustrate the transformation of a game problem to a Linear programming problem, consider a payroll matrix of m × n size. Let a_{ij} be the element in the ith row and jth column of game payroll matrix, and letting p_i be the probabilities of m strategies (i = 1, 2, …, m) for player A. Then the expected gains for player A for each of B's strategies will be

$$\sum_{i=1}^{n} p_i a_{ij}\,, j = 1, 2, \dots n \qquad \dots \qquad (1)$$

The aim of player A is to select asset of strategies with probability p_i(i = 1, 2, …, m) on any play of game such that he can maximize his minimum expected gains. To obtain values of probability p_i, the value of the game to player A for all strategies by player B must be at least equal to V. thus to maximize the minimum expected gains, it is necessary that

$$\left.\begin{aligned} a_{11}p_1 + a_{12}p_2 + \dots . a_{m1}p_m &\geq V \\ a_{12}p_1 + a_{22}p_2 + \dots . a_{m2}p_m &\geq V \\ &\vdots \\ a_{1n}p_1 + a_{2n}p_2 + \dots . a_{mn}p_m &\geq V \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (2)$$

Where, $p_1 + p_2 + \dots + p_m = 1$; $p_i \geq 0$ for all i.

Dividing both sides of the m inequalities and equation by V, the division is valid as long as V > 0. In case V < 0, the direction of the inequality constraints must be reserved. But if V = 0, division would be meaningless. In this case, a constant can be added to all entries of the matrix ensuring that the value of the game (V) for the revised matrix becomes more than zero. After optimal solution is obtained, the true value of the game is obtained by subtracting the same constant value. Let $\frac{p_i}{V} = x_i, (\geq 0)$. Then we have

$$\left.\begin{aligned} a_{11}\frac{p_1}{V} + a_{21}\frac{p_2}{V} + \dots . a_{m1}\frac{p_m}{V} &\geq 1 \\ a_{12}\frac{p_1}{V} + a_{22}\frac{p_2}{V} + \dots . a_{m2}\frac{p_m}{V} &\geq 1 \\ &\vdots \\ a_{1n}\frac{p_1}{V} + a_{2n}\frac{p_2}{V} + \dots . a_{mn}\frac{p_m}{V} &\geq 1 \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (3)$$

where $\frac{p_1}{V} + \frac{p_2}{V} + \dots + \frac{p_m}{V} = 1$.

Since the objective of player A is to maximize the value of the game, V which is equivalent to minimizing $\frac{1}{V}$, the resulting linear programming problem can be stated as

$$Minimize\ Z_p\left(=\frac{1}{V}\right) = x_1 + x_2 + \dots + x_n$$

Subject to the constraints:

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots . a_{m1}x_m &\geq 1 \\ a_{12}x_1 + a_{22}x_2 + \dots . a_{m2}x_m &\geq 1 \\ &\vdots \\ a_{1n}x_1 + a_{2n}x_2 + \dots . a_{mn}x_m &\geq 1 \\ x_i = \frac{p_i}{V} \geq 0; i &= 1, 2, \dots, m \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (4)$$

Similarly, player B has a similar problem with the inequalities of the constraints reversed, i.e. minimize the expected loss. Since minimizing of V is equivalent to maximizing $\frac{1}{V}$, therefore, the resulting linear programming problem can be stated as:

Maximize $Z_q \left(= \frac{1}{V}\right) = y_1 + y_2 + \ldots + y_n$

Subject to the constraints

$$\left.\begin{aligned} a_{11}y_1 + a_{12}y_2 + \ldots + a_{1n}y_n &\leq 1 \\ a_{12}y_1 + a_{22}y_2 + \ldots + a_{2b}y_n &\leq 1 \\ &\vdots \\ a_{m1}y_1 + a_{m2}y_2 + \ldots + a_{mn}y_n &\leq 1 \\ y_1, y_2, \ldots, y_n &\geq 0 \\ y_j = \frac{q_j}{V} \geq 0; j &= 1, 2, \ldots, n \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(5)$$

It may be noted that the linear programming problem of player B is the dual of linear programming problem of player A and vice versa. Therefore, solution of the dual problem can be obtained from the primal simplex table. Since for both players $Z_p = Z_q$, the expected gain to player A in the game will be exactly equal to expected loss to player B.

It should be noted that linear programming technique requires all variables to be non-negative and therefore to obtain a non-negative of value V of the game, the data of the problem, i.e. aij = 1 the payoff table should all be non-negative. If there are some negative elements in the payoff table, a constant to every element in the payoff table must be added so as to make the smallest element zero; the solution to this new game will give an optimal mixed strategy for the original game. The value of the original game then equals the value of the new game minus the constant.

3. Methodology

3.1. Population Projection of Enugu Urban in the Next Fifty Years from 2006 Census

The demographic data of population in Nigeria does not follow a uniform trend. Census figure supposed to be obtained every ten (10) years but the period from the available data the National Population Commission (NPC) shows that Enugu Urban Population rose from 3,170 in 1921 to 12,959 in 1931, 62,764 in 1953 to 138,874 in 1964; 385,735 in 1983. The census figures of 407,756 in 1991 to 722,664 in 2006. However, there is a sporadic increase in population of Enugu Urban due to religious, ethnic and political crises in different parts of the country especially in Northern Nigeria. Most people of Igbo extraction living in the North and West decided to relocate to Enugu as a safe choice. This is because of its position as the capital of the former Eastern region of Nigeria. This makes it difficult to forecast the population growth rate based on decades estimation.

Therefore, Geometric method using the method of assumed growth rate as adopted at 17% per decade for fast growing city like Enugu with 2006 population figure as a base year was used for the estimation.

$$Using\ \ the\ formular, P_n = P_o\left(1 + \frac{r}{100}\right)^n \quad \ldots\ldots\ (6)$$

Where P_o= Initial population i.e. the population at the end of last known census

P_n = Future population after n decades

r = Assumed growth rate (%)

n = Number of decades

It is pertinent here, to forecast the population of Enugu Urban in year 2020 and project to year 2056 with 2006 population data. This will help to forecast the volume or tons of solid waste generated, with a view of planning and managing them for effective Solid Waste Management in Enugu Urban.

Referring to the formulae above, for year 2020,

$$\mathrm{P_o = P_{2006} = 722664, r = 17\%, \quad n = 1.4}$$

$$P_n = P_{2020} = 722664, \left(1 + \frac{17}{100}\right)^{1.4} = 722664\ (1.17)^{1.4} = 900{,}319$$

$$\text{for year 2056 i. e. forecasting for the next 5 decades}$$

$$P_n = P_{2056} = 722664(1.17)^5 = 1{,}584{,}403$$

Agbaeze*et al*.(2014) estimated that Enugu Urban generated 150 metric tons per day.

In order to estimate the population based on 2006

$$P_n = P_{2014} = P_{2006}\ (1.17)^{0.8} = 722664\ (1.17)^{0.8} = 819{,}380$$

Forecasting using this research model, in 2020 we have, $\frac{900319}{819{,}380} \times 150 = 165\ tons$. However, this model has not taken into account the total waste generated in Enugu urban but on the capacity of daily disposal vehicles /Equipment by Enugu State Waste Management Authority (ESWAMA). A lot of solid waste generated are lying uncollected at alternate days while those from industries, agricultural and allied institutions were not captured because their collection route did not extend to those areas. Some of these companies use their private vehicles to dispose their Solid wastes to convenient waste disposal sites or incinerate them.

Today the waste generation has increased tremendously that ESWAMA employs the services of individual private vehicles to improve the solid waste collection and disposal. The current strategy is not allowing wastes to stay long on dumpsites before collection and disposal. The information from ESWAMA Landfill site at Enugu reveals that as at date from their records, an average of 2400 tons per day was deposited at the site in 2020 but was updated to 3234 tons by March 2025.

4. Results and Discussion

4.1. Game Theory Optimization Model

The use of Game theory model, involve to formulate the matrix for the computation of waste generation in Enugu urban. The difference from the characterized wastes was classified as organic wastes. The total waste from the option separated were deducted from the total waste generated from the projected period 2056 to determine the total organic waste generated at each location. Also some quantity of wastes generated in some wastes location at Enugu East and Enugu North were separated to create the fifth location referred to as Enugu East and the former Enugu East were renamed Enugu East Central in order to have a 5 × 5 matrix for the purpose of determining the Game theory model. This resulted to the information in Table 1 with the same quantity of tons of waste generated per day in the city.

Table 1 Quantity of waste options generated at various locations in Enugu urban.

Waste locations	**Tons of available wastes generated**					
	E-waste (x_1)	Plastics (x_2)	Ceramics (x_3)	Metals (x_4)	Organic wastes (x_5)	Total wastes
Enugu South (A_1)	122	108	3	41	869	1142
Enugu East Central (A_2)	31	142	82	28	752	1035
Enugu North East (A_3)	111	107	118	59	986	1381
Enugu North Central (A_4)	138	41	60	139	779	1157
Enugu East (A_5)	19	40	61	43	432	595
Total	421	438	323	310	3818	5310

The total revenue generated of N50.575 billion per day were used to determine the benefits of each of the tons of waste generated using pro-rata adjustment as shown below.

First (1st) row: $(i).\frac{122}{1142} \times 50.575 = 5.40\ billion$ $(ii).\frac{108}{1142} \times 50.575 = 4.78\ billion$ $(iii).\frac{2}{1142} \times 50.575 = 0.09\ billion$	$(iv).\frac{41}{1142} \times 50.575 = 1.82\ billion$ $(v).\frac{869}{1142} \times 50.575 = 38.48\ billion$
Second (2nd) row: $(i).\frac{31}{1035} \times 50.575 = 1.51\ billion$ $(ii).\frac{142}{1035} \times 50.575 = 6.94\ billion$ $(iii).\frac{82}{1035} \times 50.575 = 4.01\ billion$ $(iv).\frac{28}{1035} \times 50.575 = 1.37\ billion$ $(v).\frac{752}{1035} \times 50.575 = 36.75\ billion$	Third (3rd) row: $(i).\frac{111}{1381} \times 50.575 = 4.07\ billion$ $(ii).\frac{107}{1381} \times 50.575 = 3.92\ billion$ $(iii).\frac{118}{1381} \times 50.575 = 4.32\ billion$ $(iv).\frac{59}{1381} \times 50.575 = 2.16\ billion$ $(v).\frac{986}{1381} \times 50.575 = 36.11\ billion$
Fourth (4th) row: $(i).\frac{138}{1157} \times 50.575 = 6.03\ billion$ $(ii).\frac{41}{1157} \times 50.575 = 1.79\ billion$ $(iii).\frac{60}{1157} \times 50.575 = 2.62\ billion$ $(iv).\frac{139}{1157} \times 50.575 = 6.08\ billion$ $(v).\frac{779}{1157} \times 50.575 = 34.05\ billion$	Fifth (5th) row: $(i).\frac{19}{595} \times 50.575 = 1.62\ billion$ $(ii).\frac{40}{595} \times 50.575 = 3.40\ billion$ $(iii).\frac{61}{595} \times 50.575 = 5.19\ billion$ $(iv).\frac{43}{595} \times 50.575 = 3.66\ billion$ $(v).\frac{432}{595} \times 50.575 = 36.72\ billion$

These costs are summarized in Table 2.

Table 2 Summary of costs/benefits of various waste types generated

Player A	**Player B**					**Minimum**
	B_1 (X_1)	B_2 (X_2)	B_3 (X_3)	B_4 (X_4)	B_5 (X_5)	
A_1	5.40	4.78	0.09	1.82	38.48	**0.09**
A_2	-1.51	6.94	4.01	1.37	36.75	**1.37**
A_3	4.07	3.92	4.32	2.16	36.11	**2.16**
A_4	6.03	1.79	2.62	6.08	34.05	**1.79**
A_5	1.62	3.40	5.19	3.66	36.72	**1.62**
Maximum	**6.03**	**6.94**	**5.19**	**6.08**	**38.48**	

Minimum = 2.16; Maximum = 5.19

Since there is no saddle point, the value of the game will be calculated through iteration. Also no row or column is completely dominated by the other.

So, we apply the linear programming of game theory.

Let the value of the game = V and q_1, q_2, q_3, q_4, q_5 be the probabilities of selecting the strategies B_1, B_2, B_3, B_4, B_5 respectively.

The objectives/benefits is to maximize value of the wastes which will be generated from Table 2.

$$\left\{\begin{array}{c} 5.40q_1 + 4.78q_2 + 0.09q_3 + 1.82q_4 + 38.48q_5 \leq V \\ 1.51q_1 + 6.94q_2 + 4.01q_3 + 1.37q_4 + 36.75q_5 \leq V \\ 4.07q_1 + 3.92q_2 + 4.32q_3 + 2.16q_4 + 36.11q_5 \leq V \\ 6.03q_1 + 1.79q_2 + 2.62q_3 + 6.08q_4 + 34.05q_5 \leq V \\ 1.62q_1 + 3.4q_2 + 5.19q_3 + 3.66q_4 + 36.72q_5 \leq V \\ q_1 + q_2 + q_3 + q_4 + q_5 = 1\ (Probabilitycondition) \end{array}\right\} \ldots\ldots\ldots \quad (7)$$

Divide Equations 7 through by V, we have;

$$\left\{\begin{array}{c} 5.40q_1/V + 4.78q_2/V + 0.09q_3/V + 1.82q_4/V + 38.48q_5/V \leq 1 \\ 1.51q_1/V + 6.94q_2/V + 4.01q_3/V + 1.37q_4/V + 36.75q_5/V \leq 1 \\ 4.07q_1/V + 3.92q_2/V + 4.32q_3/V + 2.16q_4/V + 36.11q_5/V \leq 1 \\ 6.03q_1/V + 1.79q_2/V + 2.62q_3/V + 6.08q_4/V + 34.05q_5/V \leq 1 \\ 1.62q_1/V + 3.4q_2/V + 5.19q_3/V + 3.66q_4/V + 36.72q_5/V \leq 1 \\ q_1 */V + q_2/V + q_3/V + q_4/V + q_5/V = 1 \end{array}\right\} \ldots (8)$$

$$\frac{letq_1}{V} = \frac{x_1, q_2}{V} = x_2, \frac{q_3}{V} = x_3, \frac{q_4}{V} = x_4 and, \frac{q_5}{V} = x_5 \qquad \ldots\ldots\ldots \quad (9)$$

The values in equations 8 and 9 are converted into a linear programming problem as;

$MaximizeZ_p = \left(\frac{1}{V}\right) = x_1 + x_2 + x_3 + x_4 + x_5$

Subject to:

$$\left.\begin{array}{l} 5.4x_1 + 4.78x_2 + 0.09x_3 + 1.82x_4 + 38.48x_5 \leq 1 \\ 1.51x_1 + 6.94x_2 + 4.01x_3 + 1.37x_4 + 36.75x_5 \leq 1 \\ 4.07x_1 + 3.92x_2 + 4.32x_3 + 2.16x_4 + 36.11x_5 \leq 1 \\ 6.03x_1 + 1.97x_2 + 2.62x_3 + 6.08x_4 + 34.05x_5 \leq 1 \\ 1.62x_1 + 3.40x_2 + 5.19x_3 + 3.66x_4 + 36.72x_5 \leq 1 \end{array}\right\} \quad (10)$$

Since we have more than two variables, the simplex method of linear programming is used to solve the problem by introducing slack variables to convert the inequalities to equations which becomes;

$MaximizeZ_q = \left(\frac{1}{V}\right) = x_1 + x_2 + x_3 + x_4 + x_5 + 0S_1 + 0S_2 + 0S_3 + 0S_4 + 0S_5$

Subject to the following constraints;

$$\left.\begin{array}{l} 5.4x_1 + 4.78x_2 + 0.09x + 1.82x_4 + 38.48x_5 + S_1 + 0S_2 + 0S_3 + 0S_4 + 0S_5 = 1 \\ 1.51x_1 + 6.94x_2 + 4.01x_3 + 1.37x_4 + 36.75x_5 + 0S_1 + S_2 + 0S_3 + 0S_4 + 0S_5 = 1 \\ 4.07x_1 + 3.92x_2 + 4.32x_3 + 2.16x_4 + 36.11x_5 + 0S_1 + 0S_2 + S_3 + 0S_4 + 0S_5 = 1 \\ 6.03x_1 + 1.97x_2 + 2.62x_3 + 6.08x_4 + 34.05x_5 + 0S_1 + 0S_2 + 0S_3 + S_4 + 0S_5 = 1 \\ 1.62x_1 + 3.40x_2 + 5.19x_3 + 3.66x_4 + 36.72x_5 + 0S_1 + 0S_2 + 0S_3 + 0S_4 + S_5 = 1 \\ x_1, x_2, x_3, x_4, x_5, S_1, S_2, S_3, S_4, S_5 \geq 0 \end{array}\right\} \ldots\ldots\ldots \quad (11)$$

The equations formulated are used to solve the Simplex method by forming the initial Simplex table.

Table 3 Initial Simplex Table

Variables		x_1	x_2	x_3	x_4	**x_5**	S_1	S_2	S_3	S_4	S_5	Amount	
Basis	C_j	1	1	1	1	**1**	0	0	0	0	0		Trade ratio
S_1	0	**5.40**	**4.78**	**0.09**	**1.82**	**38.48**	**1**	**0**	**0**	**0**	**0**	**1**	$\frac{1}{38.48} = 0.026$ Take out ←
S_2	0	1.51	6.94	4.01	1.37	**36.75**	0	1	0	0	0	1	$\frac{1}{36.75} = 0.027$
S_3	0	4.07	3.92	4.32	2.16	**36.11**	0	0	1	0	0	1	$\frac{1}{36.11} = 0.028$
S_4	0	6.03	1.79	2.62	6.08	**34.05**	0	0	0	1	0	1	$\frac{1}{34.05} = 0.029$
S_5	0	1.62	3.40	5.19	3.66	**36.72**	0	0	0	0	1	1	$\frac{1}{36.72} = 0.027$
Zj		0	0	0	0	**0**	0	0	0	0	0	0	
$C_j - Z_j$		1	1	1	1	**1**	0	0	0	0	0		
						↑ Bring in							

Table 4 Second (2nd) Simplex table

Variables		**x_1**	**x_2**	**x_3**	**x_4**	**x_5**	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**		
Basis	C_j	1	1	1	1	1	0	0	0	0	0	Amount	Trade ratio
x_5	1	0.14	0.12	**0.002**	0.05	1	0.026	0	0	0	0	0.026	$\frac{0.026}{0.002} = 13$
S_2	0	-3.64	6.94	**4.01**	1.37	0	-0.96	1	0	0	0	0.045	$\frac{0.045}{3.94} = 0.011$
S_3	0	-0.99	3.92	**4.32**	2.16	0	-0.94	0	1	0	0	0.061	$\frac{0.061}{4.25} = 0.014$
S_4	0	1.26	1.79	**2.62**	6.08	0	-0.89	0	0	1	0	0.115	$\frac{0.115}{2.55} = 0.045$
S_5	0	**-3.52**	**3.40**	**5.19**	**3.66**	**0**	**-0.95**	**0**	**0**	**0**	**1**	**0.045**	$\frac{0.045}{4.24} = 0.011$ Take out ←
Z_j		0.14	0.12	0.002	0.05	1	0.026	0	0	0	0	0.026	
$C_j - Z_j$		0.36	0.88	0.998	0.95	0	-0.026	0	0	0	0		
				↑ Bring in									

Computing of values for S_2 Row

$x_1 = 1.51 - 0.14 \times 36.75 = - 3.64$
$x_2 = 6.94 - 0.12 \times 36.75 = 2.53$
$x_3 = 4.01 - 0.002 \times 36.75 = 3.94$
$x_4 = 1.37 - 0.05 \times 36.75 = 0.47$
$x_5 = 6.94 - 1 \times 36.75 = 0$
$S_1 = 0 - 0.026 \times 36.75 = - 0.96$
$S_2 = 1 - 0 \times 36.75 = 1$
$S_3 = 0 - 0 \times 36.75 = 0$
$S_4 = 0 - 0 \times 36.75 = 0$
$S_5 = 0 - 0 \times 36.75 = 0$
Amount $= S_2 = 1 - 0.026 \times 36.75 = 0.045$

Computing of values for S_3 Row

$x_1 = 4.07 - 0.14 \times 36.11 = - 3.64$
$x_2 = 3.92 - 0.12 \times 36.11 = 2.53$
$x_3 = 4.32 - 0.002 \times 36.11 = 3.94$
$x_4 = 2.16 - 0.05 \times 36.11 = 0.47$
$x_5 = 36.11 - 1 \times 36.11 = 0$
$S_1 = 0 - 0.026 \times 36.11 = - 0.94$
$S_2 = 0 - 0 \times 36.11 = 0$
$S_3 = 1 - 0 \times 36.11 = 1$
$S_4 = 0 - 0 \times 36.11 = 0$
$S_5 = 0 - 0 \times 36.11 = 0$
Amount $= S_3 = 1 - 0.026 \times 36.11 = 0.061$

Computing of values for S_4 Row

$x_1 = 6.03 - 0.14 \times 34.05 = 1.26$
$x_2 = 1.97 - 0.12 \times 34.05 = -2.12$
$x_3 = 2.62 - 0.002 \times 34.05 = 2.55$
$x_4 = 6.08 - 0.05 \times 34.05 = 4.38$
$x_5 = 34.05 - 1 \times 34.05 = 0$
$S_1 = 0 - 0.026 \times 34.05 = -0.89$
$S_2 = 0 - 0 \times 34.05 = 0$
$S_3 = 0 - 0 \times 34.05 = 0$
$S_4 = 1 - 0 \times 34.05 = 1$
$S_5 = 0 - 0 \times 34.05 = 0$
Amount $= S_4 = 1 - 0.026 \times 34.05 = 0.115$

Computing of values for S_5 Row

$x_1 = 1.62 - 0.14 \times 36.72 = -3.52$
$x_2 = 3.40 - 0.12 \times 36.72 = -1.01$
$x_3 = 5.19 - 0.002 \times 36.72 = 4.24$
$x_4 = 3.66 - 0.05 \times 36.72 = 1.82$
$x_5 = 36.72 - 1 \times 36.72 = 0$
$S_1 = 0 - 0.026 \times 36.72 = -0.95$
$S_2 = 0 - 0 \times 36.72 = 0$
$S_3 = 0 - 0 \times 36.72 = 0$
$S_4 = 0 - 0 \times 36.72 = 0$
$S_5 = 1 - 0 \times 36.72 = 1$
Amount $= S_5 = 1 - 0.026 \times 36.72 = 0.045$

Table 5 Third (3rd) Simplex table

Variables		x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5		
Basis	C_j	1	1	1	1	1	0	0	0	0	0	Amount	Trade ratio
x_3	1	**-0.83**	-0.23	1	0.43	0	-0.22	0	0	0	0.24	0.011	$\frac{0.011}{-0.83} = 0.013$
x_5	1	**0.14**	0.12	0	0.05	1	-0.026	0	0	0	-0.005	0.026	$\frac{0.026}{0.14} = 0.186$
S_2	0	**-0.37**	3.44	0	-1.22	0	-0.09	1	0	0	-0.95	0.002	$\frac{0.002}{0.37} = 0.0054$
S_3	0	**2.54**	**0.57**	**0**	**-1.48**	**0**	**-0.005**	**0**	**1**	**0**	**-1.02**	**0.014**	$\frac{0.014}{2.54} = 0.0055$ Take out ←
S_4	0	**3.38**	-1.53	0	3.28	0	-0.33	0	0	1	-0.61	0.087	$\frac{0.087}{3.38} = 0.0257$
Z_j		-0.69	-0.11	1	0.48	1	-0.19	0	0	0	0.24	0.037	
$C_j - Z_j$		1.69	1.11	0	0.52	0	0.19	0	0	0	-0.24		

↑ Bring in (under x_1)

Computing of values for X_5 Row x_1 = 0.14 – (-0.83 × 0.002) = 1.14 x_2 = 0.12 – (-0.23 × 0.002) = 0.12 x_3 = 0.002 – 1 × 0.002 = 0 x_4 = 0.05 – 0.43 × 0.002 = 0.05 x_5 = 1 – 0 × 0.002 = 1 S_1 = – 0.026 – (-0.22 × 0.002) = - 0.026 S_2 = 0 – 0 × 0.002 = 0 S_3 = 0 – 0 × 0.002 = 0 S_4 = 0 – 0 × 0.002 = 0 S_5 = 0 – 0.24 × 0.002 = -0.0005 Amount = 0.026 – 0.011× 0.002 = 0.026	Computing of values for S_2 Row x_1 = -3.64 – (-0.83 × 3.94) = -0.37 x_2 = 0.12 – (-0.23 × 3.94) = 3.44 x_3 = 3.94 – 1 × 3.94 = 0 x_4 = 0.47 – 0.43 × 3.94 = -1.22 x_5 = 0 – 0 × 3.94 = 0 S_1 = – 0.96 – (-0.22 × 3.94) = - 0.09 S_2 = 1 – 0 × 3.94 = 1 S_3 = 0 – 0 × 3.94 = 0 S_4 = 0 – 0 × 3.94 = 0 S_5 = 0 – 0.24 × 3.94 = -0.95 Amount = 0.045 – 0.011× 3.94 = 0.002
Computing of values for S_3 Row x_1 = -0.99 – (-0.83 × 4.25) = 2.54 x_2 = 0.41 – (-0.23 × 4.25) = 0.57 x_3 = 4.25 – 1 × 4.25 = 0 x_4 = 0.35 – 0.43 × 4.25 = -1.48 x_5 = 0 – 0 × 4.25 = 0 S_1 = – 0.94 – (-0.22 × 4.25) = - 0.05 S_2 = 0 – 0 × 4.25 = 0 S_3 = 1 – 0 × 4.25 = 1 S_4 = 0 – 0 × 4.25 = 0 S_5 = 0 – 0.24 × 4.25 = -1.02 Amount = 0.061 – 0.011× 4.25 = 0.014	Computing of values for S_4 Row x_1 = 1.26 – (-0.83 × 2.55) = 3.38 x_2 = -2.12 – (-0.23 × 2.55) = -1.53 x_3 = 2.55 – 1 × 2.55 = 0 x_4 = 4.38 – 0.43 × 2.55 = 3.28 x_5 = 0 – 0 × 2.55 = 0 S_1 = – 0.89 – (-0.22 × 2.55) = - 0.33 S_2 = 0 – 0 × 2.55 = 0 S_3 = 0 – 0 × 2.55 = 0 S_4 = 1 – 0 × 2.55 = 1 S_5 = 0 – 0.24 × 2.55 = -0.61 Amount = 0.061 – 0.011× 2.55 = 0.087

Table 6 Fourth (4th) Simplex table

Variables		x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5		
Basis	C_j	1	1	1	1	1	0	0	0	0	0	Amount	Trade ratio
x_1	1	1	0.22	0	**-0.58**	0	-0.02	0	0.39	0	-0.40	0.0055	$\frac{0.0055}{-0.58} = 0.009$
x_3	1	0	-0.05	1	**0.05**	0	-0.22	0	0.32	0	-0.09	0.016	$\frac{0.016}{0.05} = 0.32$
x_5	1	0	0.09	0	**0.13**	1	0.026	1	-0.05	0	0.06	0.025	$\frac{0.025}{0.13} = 0.192$
S_3	0	0	3.52	0	**-1.43**	0	-0.09	0	0.14	0	-1.10	0.004	$\frac{0.004}{-1.43} = 0.0028$
S_4	0	**0**	**-2.27**	**0**	**5.24**	**0**	**-0.32**	**0**	**-1.32**	**1**	**0.74**	**0.068**	$\frac{0.068}{5.24} = 0.013$ Take out ←
Z_j		1	0.26	1	-0.4	1	-0.20	0	0.66	0	-0.53	0.047	
$C_j - Z_j$		0	0.74	0	1.4	0	0.20	0	-0.66	0	0.53		

↑ Bring in

Computing of values for X_3 Row x_1 = -0.83 – (1 × – 0.83) = 0 x_2 = -0.23 – 0.22 × –0.83 = -0.047 x_3 = 1–0 × –0.83 = 1 x_4 = –0.43– (–0.58× –0.83) = 0.051 x_5 = 0 – 0 × –0.83 = 0 S_1 = – 0.22 – (-0.002 × –0.83) = - 0.22 S_2 = 1 – 0 × –0.83 = 0 S_3 = 0 – 0.39 × –0.83 = 0.32 S_4 = 0 – 0 × –0.83 = 0 S_5 = 0 – 0.24 –(–0.40 ×–0.83) = -0.09 Amount = 0.011 – 0.0055× –0.83 = 0.016.	Computing of values for S_2 Row x_1 = -0.37 – (1 × – 0.37) = 0 x_2 = 3.44 – 0.22 × –0.37 = 3.52 x_3 = 0–0 × –0.37 = 0 x_4 = –1.22 – (–0.58× –0.37) = – 1.43 x_5 = 0 – 0 × –0.37 = 0 S_1 = – 0.09 – (– 0.002 × –0.37) = – 0.091 S_2 = 1 – 0 × –0.37 = 1 S_3 = 0 – 0.39 × –0.37 = 0.14 S_4 = 0 – 0 × –0.37 = 0 S_5 = 0 – 0.95 –(–0.40 ×–0.37) = -1.10 Amount = 0.002 – 0.0055× –0.37 = 0.004.
Computing of values for X_5 Row x_1 = 0.14 – 1 × – 0.14 = 0 x_2 = 0.12 – 0.22 × 0.14 = 0.09 x_3 = 0 – 0 × 0.14 = 0 x_4 = 0.05 – (–0.58× 0.14) = 0.13 x_5 = 1 – 0 × 0.14 = 1 S_1 = 0.026 – (– 0.002 × 0.14) = – 0.026 S_2 = 0 – 0 × 0.14 = 0 S_3 = 0 – 0.39 × 0.14 = 0.05 S_4 = 0 – 0 × 0.14 = 0 S_5 = – 0.0005 –(–0.40 × 0.14) = 0.06 Amount = 0.026 – 0.0055× 0.14 = 0.025.	Computing of values for S_4 Row x_1 = 3.38 – 1 × – 3.38 = 0 x_2 = –1.53 – 0.22 × 3.38 = –2.27 x_3 = 0 – 0 × 3.38 = 0 x_4 = 3.28 – (–0.58× 3.38) = 5.24 x_5 = 0 – 0 × 3.38 = 0 S_1 = – 0.33 – (– 0.002 × 3.38) = – 0.32 S_2 = 0 – 0 × 3.38 = 0 S_3 = 0 – 0.39 × 3.38 = –1.32 S_4 = 1 – 0 × 3.38 = 1 S_5 = – 0.61 –(–0.40 × 3.38) = 0.74 Amount = 0.087 – 0.0055× 3.38 = 0.068.

Table 7 Fifth (5th) Simplex table

Variables		**x_1**	**x_2**	**x_3**	**x_4**	**x_5**	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**		
Basis	C_j	1	1	1	1	1	0	0	0	0	0	Amount	Trade ratio
x_4	1	1	**-0.43**	0	1	0	-0.06	0	-0.25	0.19	0.14	0.013	$\frac{0.013}{-0.43} = -0.009$
x_1	1	0	**-0.03**	0	0	0	0.03	0	0.25	0.11	-0.32	0.013	$\frac{0.013}{-0.03} = -0.455$
x_3	1	0	**-0.03**	1	0	0	-0.22	0	0.33	-0.01	-0.1	0.015	$\frac{0.015}{-0.03} = -0.5$
x_5	1	0	**0.15**	0	0	1	0.03	0	0.02	-0.03	0.04	0.023	$\frac{0.023}{0.15} = 0.153$
S_2	0	**0**	**2.91**	**0**	**0**	**0**	**-0.18**	**1**	**-0.22**	**0.27**	**-0.90**	**0.023**	$\frac{0.023}{2.91} = 0.0079$ Take ← out
Z_j		1	-0.34	1	1	1	-0.22	0	0.35	0.26	-0.24	0.064	
$C_j - Z_j$		0	1.34	0	0	0	0.22	0	-0.35	-0.26	0.24		

↑ Bring in

Computing of values for X_1 Row	Computing of values for X_5 Row
$x_1 = 1 - 0 \times - 0.58 = 1$	$x_1 = 0 - 0 \times 0.13 = 0$
$x_2 = -0.22 - (-0.43 \times -0.58) = 0.03$	$x_2 = 0.09 - (-0.43 \times 0.13) = 0.15$
$x_3 = 0 - 0 \times -0.58 = 0$	$x_3 = 0 - 0 \times 0.13 = 0$
$x_4 = 0.58 - 1 \times -0.58 = 0$	$x_4 = 0.13 - 1 \times 0.13 = 0$
$x_5 = 0 - 0 \times -0.58 = 0$	$x_5 = 1 - 0 \times 0.13 = 1$
$S_1 = - 0.002 - 0.06 \times -0.58 = 0.03$	$S_1 = 0.026 -(- 0.06 \times 0.13) = 0.03$
$S_2 = 0 - 0 \times -0.58 = 0$	$S_2 = 0 - 0 \times 0.13 = 0$
$S_3 = 0.39 - (- 0.25 \times -0.58) = 0.25$	$S_3 = -0.05 - (- 0.25 \times 0.13) = 0.02$
$S_4 = 0 - 0.19 \times -0.58 = 0.11$	$S_4 = 0 - 0.19 \times 0.13 = - 0.025$
$S_5 = - 0.40 - 0.14 \times -0.58 = -0.32$	$S_5 = - 0.06 - 0.14 \times 0.13 = 0.042$
Amount $= 0.0055 - 0.013 \times -0.58 = 0.013$.	Amount $= 0.025 - 0.013 \times 0.13 = 0.023$.
Computing of values for X_3 Row	Computing of values for S_2 Row
$x_1 = 0 - 0 \times 0.05 = 0$	$x_1 = 0 - 0 \times - 1.43 = 0$
$x_2 = -0.05 - (-0.43 \times 0.05) = -0.03$	$x_2 = 3.52 - (-0.43 \times - 1.43) = 2.91$
$x_3 = 1 - 0 \times 0.05 = 1$	$x_3 = 0 - 0 \times - 1.43 = 0$
$x_4 = 0.05 - 1 \times 0.05 = 0$	$x_4 = 1.43 - 1 \times - 1.43 = 0$
$x_5 = 0 - 0 \times 0.05 = 0$	$x_5 = 0 - 0 \times - 1.43 = 0$
$S_1 = -0.22 -(- 0.06 \times 0.05) = -0.22$	$S_1 = - 0.09 -(- 0.06 \times - 1.43) = 0.18$
$S_2 = 0 - 0 \times 0.05 = 0$	$S_2 = 1 - 0 \times - 1.43 = 1$
$S_3 = 0.32 - (- 0.25 \times 0.05) = 0.33$	$S_3 = 0.14 - (- 0.25 \times - 1.43) = - 0.22$
$S_4 = 0 - 0.19 \times 0.05 = - 0.010$	$S_4 = 0 - 0.19 \times - 1.43 = - 0.25$
$S_5 = - 0.00 - 0.14 \times 0.05 = -0.10$	$S_5 = - 1.10 - 0.14 \times -1.43 = 0.042$
Amount $= 0.016 - 0.013 \times 0.05 = 0.015$.	Amount $= 0.004 - 0.013 \times - 1.43 = 0.023$.

Table 8 Sixth (6th) Simplex Table (Optimal Solution)

Variables		**x_1**	**x_2**	**x_3**	**x_4**	**x_5**	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**	
Basis	C_j	1	1	1	1	1	0	0	0	0	0	Amount
x_2	1	0	1	0	0	0	-0.06	0.34	-0.08	0.09	-0.31	**0.008**
x_4	1	1	0	0	1	0	-0.03	0.15	0.22	0.23	0.07	**0.0164**
x_1	1	0	0	0	0	0	0.03	0.01	0.25	0.11	-0.33	**0.0132**
x_3	1	0	0	1	0	0	-0.22	0.01	0.33	0.01	0.33	**0.0152**
x_5	1	0	0	0	0	1	0.04	-0.05	0.03	-0.04	-0.09	**0.0218**
Z_j		1	1	1	1	1	-0.24	0.46	0.75	0.4	-0.92	**0.0746**
$C_j - Z_j$		0	0	0	0	0	0.24	-0.46	-0.75	-0.4	0.92	

Computing of values for X_4 Row	Computing of values for X_1 Row
$x_1 = 1 - 0 \times - 0.43 = 1$	$x_1 = 0 - 0 \times - 0.03 = 0$
$x_2 = -0.43 - 1 \times -0.43 = 0$	$x_2 = -0.03 - 1 \times -0.03 = 0$
$x_3 = 0 - 0 \times -0.43 = 0$	$x_3 = 0 - 0 \times -0.03 = 0$
$x_4 = 1 - 0 \times -0.43 = 1$	$x_4 = 0 - 0 \times -0.03 = 0$

$x_5 = 0 – 0 \times –0.43 = 0$ $S_1 = – 0.006 – 0.06 \times –0.43 = 0.03$ $S_2 = 0 – 0.34 \times –0.43 = 0.15$ $S_3 = – 0.25 – (– 0.25 \times –0.43) = 0.22$ $S_4 = 0.19 – 0.09 \times –0.43 = 0.23$ $S_5 = – 0.14 –(–0.31 \times –0.43 = 0.007$ Amount= 0.013 – 0.008 × –0.43 = 0.0164.	$x_5 = 0 – 0 \times –0.03 = 0$ $S_1 = – 0.03 – (–0.06 \times –0.03) = 0.028$ $S_2 = 0 – 0.34 \times –0.03 = 0.01$ $S_3 = 0.25 – (– 0.08 \times –0.03) = 0.25$ $S_4 = 0.11 – 0.09 \times –0.03 = 0.11$ $S_5 = – 0.32 – (–0.31 \times –0.03) = -0.33$ Amount= 0.013 – 0.003 × –0.03 = 0.0132.
Computing of values for X_3 Row $x_1 = 0 – 0 \times – 0.03 = 0$ $x_2 = –0.03 \times 1 \times – 0.03 = 0$ $x_3 = 1 – 0 \times –0.03 = 1$ $x_4 = 0 – 0 \times –0.03 = 0$ $x_5 = 0 – 0 \times –0.03 = 0$ $S_1 = –0.22 –(– 0.06 \times 0.13) = –0.22$ $S_2 = 0 – 0.34 \times –0.03 = 0.01$ $S_3 = –0.33 – (– 0.08 \times –0.03) = 0.33$ $S_4 = –0.01 – 0.09 \times –0.03 = – 0.007$ $S_5 = – 0.1 –(– 0.31 \times –0.03) = –0.38$ Amount = 0.015–0.008 × –0.03 = 0.0152.	Computing of values for X_5 Row $x_1 = 0 – 0 \times – 0.15 = 0$ $x_2 = 0.15 – 1 \times 0.15 = 0$ $x_3 = 0 – 0 \times 0.15 = 0$ $x_4 = 0 – 0 \times 0.15 = 0$ $x_5 = 1 – 0 \times 0.15 = 1$ $S_1 = – 0.03 – (–0.06 \times 0.15) = 0.04$ $S_2 = 0 – 0.34 \times 0.15 = – 0.05$ $S_3 = 0.02 – (– 0.08 \times 0.15) = 0.03$ $S_4 = –0.03 – 0.09 \times 0.15 = 0.04$ $S_5 = 0.04 –(–0.31 \times 0.15) = 0.09$ Amount= 0.023 – 0.008 × 0.15 = 0.0218.

The optimal solution from the game model simplex method of linear programming in Table 8 shows that x_1 = 0.0132, x_2 = 0.008, x_3 = 0.0152, x_4 = 0.0164 and x_5 = 0.0218. The expected value of the game obtained from the relation $Z_q = \frac{1}{V}$ = 0.0746.

$$Therefore, \quad V = \frac{1}{0.0746.} = 13.404826 \approx 13.405$$

Converting these solution values back into original variables, we have

$$From, x_n = \frac{q_n}{V} \Rightarrow q_n = x_n \times V; substituting the values,$$

$$q_1 = x_1 \times V = 0.0132 \times 13.405 = 0.176946 \approx 0.18$$

$$q_2 = x_2 \times V = 0.008 \times 13.405 = 0.107240 \approx 0.11$$

$$q_3 = x_3 \times V = 0.0152 \times 13.405 = 0.203756 \approx 0.20$$

$$q_4 = x_4 \times V = 0.0164 \times 13.405 = 0.219842 \approx 0.22$$

$$q_5 = x_5 \times V = 0.0218 \times 13.405 = 0.292229 \approx 0.29$$

Total = 1.000013 ≈ 1.00.

The probability of selecting the various categories of wastes generated.

(i). tons of optimal quantity of e-waste (q_1) = 0.18
(ii). tons of optimal quantity of plastics (q_2) = 0.11
(iii). tons of optimal quantity of ceramics (q_3) = 0.20
(iv). tons of optimal quantity of metals (q_4) = 0.22
(v). tons of optimal quantity of organic wastes (q_5) = 0.29.

Table 9 Weighted Average tons of Waste Generated

Categories of wastes	Total generated in tons	Probability	Weighted average in tons	Total value in tons
e-waste (x_1)	421	0.18	421 × 0.18	75.78
Plastics (x_2)	438	0.11	438 × 0.11	48.18
Ceramics (x_3)	323	0.20	323 × 0.20	64.60
Metal (x_4)	310	0.22	310 × 0.22	68.20
Organic waste (x_5)	3818	0.29	3818 × 0.29	1107.22
Total	5310	1.00		1363.93

Therefore the financial benefit under the worst condition will be 1363.93 × 13.405 = ₦18284.15 billion or ₦18.284 trillion.

Alternatively, the probabilities can be applied to value of the game and the cost multiplied by the quantity of each tons of waste generated to get the total benefit, we have;

Table 10 Total Revenue Generated in Billions of Naira

Types of wastes (B_1)	Prob. (B_2)	Cost in billions (B_3)	Total weight in tons (B_4)	Total revenue generated in billions ($B_3 \times B_4$)
e-waste (x_1)	0.18	0.18 × 13.405 = ₦2.413	421	1015.873
Plastics (x_2)	0.11	0.11 × 13.405 = ₦1.475	438	646.05
Ceramics (X_3)	0.20	0.20 × 13.405 = ₦2.681	323	865.963
Metal (X_4)	0.22	0.22 × 13.405 = ₦2.949	310	94.19
Organic waste (x_5)	0.29	0.29 × 13.405 = ₦3.881	3818	14, 840.566
		Total value	=	₦18, 282.642

The Total revenue is the same which amounts to ₦18, 284 trillion.

5. Conclusion

It is a clear fact that the waste management practice in Enugu urban is unsatisfactory and good strategies/measures needs to be employed to salvage the situation. The two different waste management options that must be combined intelligently in a way as to reduce the environmental, and social impact of wastes are improving the aesthetic of the city and living conditions of residents within the area. The combined option of integrated solid waste management and system approach should be used for the assessment of the competing options.

Waste recycling will be a profitable venture because it will help to grow the economy of Enugu urban. It will help to create employment among the youths and increase the standard of living of the people.

Source reduction backed by effective legislation will encourage companies to use materials that are less hazardous for packaging their products thereby reducing waste and encourage recycling of packages for manufactured products.

The integrated solid waste management above will solve the problem of solid waste through;

- **Compaction of solid waste:** there is compaction vehicle already in use by ESWAMA but some of the trucks have broken down due to lack of maintenance so increasing the number of compaction vehicle will reduce the number of trips of delivery vehicles.

- **Estimation of methane gas from waste generation:** the recovery products from landfills is methane which is produced by degradable wastes. It is useful in estimating the recovery value of methane from landfill emission. Methane production from landfills is estimated on chemical composition of solid wastes from Enugu urban. This product if harnessed will be another source of revenue.

- **The biogas recovery value:** $1m^3$ of biogas can generate 1.2 Kw/hr of electricity or $200m^3$ of biogas can generate $1.25 \times 180 = 225$ Kw/hr of electricity. This will generate a lot of revenue from biodegradable wastes for the government.

Recommendation

It is in this regards that this study here suggest the following recommendations;

- Environmental education program can be enhanced through public participation as it affects solid waste management. Grassroots enlightenment campaigns through the chiefs and community leaders with radio, television and print media will create more awareness.
- The involvement, participation and cooperation of local communities and the government will go a long way to establish effective solid wastes management in Enugu.
- Public, Private, Partnership (PPP) in Solid Waste Planning will ensure efficiency and better Management of the environment.
- Good access road should be provided through all the streets within the municipal to aid accessibility of the waste collection trucks to dumpsters and dump sites location.
- Environmental legislation should be enacted by the government to encourage source reduction of wastes, environmental sanitation and other associated matters.
- Competent penal institution should be established for proper enforcement and implementation of environmental laws.
- The procurement of additional compaction vehicles will facilitate and ease the problem of collection to disposal location for use in creating wealth for the state.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest.

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