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The order of approximation functions of several variables α - singular integrals

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Abstract

In solving the problem of determining saturation classes, a number of important results were obtained by P.I. Romanovsky, I.P. Natanson, D.K. Fadeev, B.I. Kornblum, F. Kharshiladze, A. Turetsky, P.L. Butzer, R. Nessel, R.G. Mamedov and others. Using the Fourier transform method, Butzer [1-2], R.G. Mamedov [4] and others determined the order and saturation class of various singular integrals and linear operators in the space $L_p(-\infty,\infty)$ $(p \geq 1)$.

The main results obtained in recent years by various authors on the solution of the saturation problem are described in detail in the monographs of R.G. Mamedov [4] and Butzer-Berenz [3].

In this paper, the order and class of saturation are determined

α -singular integrals of general form in the metric of the space $L^p(R_n)$. The results obtained are applied to determining the order and saturation class of a specific singular integral.

Keywords: Singular Integral; Saturation Class; Spaces; Asymptotics A; Approximations; Kernel; Transformations

1. Introduction

Let $R_n - n$ dimensional Euclidean space and let

$$\lambda = (\lambda_1,\ldots,\lambda_n),\ t = (t_1,\ldots,t_n) \text{ And } K_\lambda(t) = \prod_{e=1}^{n} K_{e,\lambda_e}(t_e)$$

where $K_{e,\lambda_e}(t_e),\ (t_e \in R_1,\ \lambda_e > 0,\ 1 \leq e \leq n)$ are one-dimensional kernels satisfying the conditions

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} K_{e,\lambda_e}(t_e)dt_e = 1,\ \left\| K_{e,\lambda_e}(t_e) \right\|_{L(R_1)} \leq M_e < \infty \text{ And}$$

$$\lim_{\lambda_e \to \infty} \int_{\substack{|t_e| \geq \delta \\ (\delta > 0)}} \left| K_{e,\lambda_e}(t_e) \right| dt_e = 0 \qquad (1)$$

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for everyone $1 \le e \le n$.

Let us consider for each $1 \le e \le n$, α a singular integral of the form:

$$Q_{e,\lambda_e}^{(\alpha)}(f;x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sum_{s_e=1}^{\infty} (-1)^{s_e-1} \binom{\alpha}{\bar{s}_e} f(x_1,\dots,x_{e-1},x_e - s_e t_e, x_{e+1},\dots,x_n) K_{e,\lambda_e}(t_e) dt_e \quad (2)$$

where $\alpha > 0$ is any real number.

In this paper, approximations of functions are considered. $f(x) \in L^p(R_n)$ α - singular integrals of general form:

$$Q_{\lambda}^{(\alpha)}(f;x) = \frac{1}{\left(\sqrt{2\pi}\right)^n} \int_{R_n} \left\{ \sum_{s_1,\dots,s_n=1}^{\infty} \left[\prod_{e=1}^{n} (-1)^{s_e-1} \binom{\alpha}{s_e} \right] f(x_1 - s_1 t_1, \dots, x_n - s_n t_n) \right\} \cdot$$

$$\cdot \left[\prod_{e=1}^{n} K_{e,\lambda_e}(t_e) \right] dt_1 \dots dt_n ; K_{\lambda}(t) = \prod_{e=1}^{n} K_{e,\lambda_e}(t_e) \quad (3)$$

where one-dimensional kernels satisfy conditions (1).

Note that if $f(x) \in L^p(R_n)$ $1 \le p < \infty$ and the kernel $K_{e,\lambda_e}(t_e)$ satisfies conditions (1), then the singular integral (3) exists almost everywhere on R_n and the following relations are valid:

$$\text{(A)} \left\| Q_{\lambda}^{(\alpha)}(f;x) \right\|_{L^p(R_n)} \le C_2 \left\| f(x) \right\|_{L^p(R_n)} \cdot \prod_{e=1}^{n} \left\| K_{e,\lambda_e}(t_e) \right\|_{L(R_1)}$$

$$\text{(b)} \lim_{\substack{\lambda_1 \to \infty \\ \dots\dots \\ \lambda_n \to \infty}} \left\| Q_{\lambda}^{(\alpha)}(f;x) - f(x) \right\|_{L^p(R_n)} = 0$$

$$\text{(With)} \lim_{\substack{\lambda_\mu \to \infty \\ (\mu \ne e)}} \left\| Q_{\lambda}^{(\alpha)}(f;x) - f(x) \right\|_{L^p(R_n)} = \left\| Q_{e,\lambda_e}^{(\alpha)}(f;x) - f(x) \right\|_{L^p(R_n)}$$

for each $1 \le \mu \le n$ and $\lim\limits_{\substack{\lambda_\mu \to \infty \\ (\mu \ne e)}}$ means that $\lambda_\mu \to \infty$ for each $1 \le \mu \le n$ ($\mu \ne e$).

$$\text{(d)} \left\| f(x) - Q_{\lambda}^{(\alpha)}(f;x) \right\|_{L^p(R_n)} \le \sum_{\mu=1}^{n-1} \left\{ \sum_{S_{\mu+1},\dots S_n=1}^{\infty} \left[\prod_{e=\mu+1}^{n} \binom{\alpha}{\bar{S}_e} \right] \cdot \right.$$

$$\left. \cdot \prod_{e=\mu+1}^{n} \left\| K_{e,\lambda_e}(t_e) \right\|_{L(R_1)} \cdot \left\| f(x) - Q_{\mu,\lambda_\mu}^{(\alpha)}(f;x) \right\|_{L^p(R_n)} \right\} +$$

$$+ \left\| f(x) - Q_{n,\lambda_n}^{(\alpha)}(f;x) \right\|_{L^p(R_n)}$$

In what follows we will assume that

$$G_{e,\lambda_e}^{(\alpha)}(u_e) = \sum_{s_e=1}^{\infty} (-1)^{s_e-1} \binom{\alpha}{\bar{S}_e} K_{e,\lambda_e}^{\wedge}(u_e s_e),$$

for $\lambda_e > 0$ each $1 \le e \le n$, where $K_{e,\lambda_e}^{\wedge}(u_e)$ is the Fourier transform of the function $K_{e,\lambda_e}(t_e)$.

Let us denote by F the set of all infinitely differentiable functions with compact support.

Let us introduce a class of functions

$$M_F^e(\psi) \equiv \{\psi(x) \in F,\ \eta_e(u_e)\psi^{\wedge}(u) = r_{\psi}^{\wedge}(u) \text{ for some } r_{\psi}(x) \in F, \eta_e(u_e) \ne 0,\ 1 \le e \le n\}.$$

Theorem. Let $f(x) \in L^p(R_n)$ ($1 \le p < \infty$) and one-dimensional kernels $K_{e,\lambda_e}(t_e)$ ($t_e \in R_1$, $\lambda_e > 0, e = \overline{1,n}$) of the singular integral (2) be such that the functions

$$\beta_{\lambda_e}^{(\alpha)}(u_e) = \frac{1 - G_{e,\lambda_e}^{(\alpha)}(u_e)}{\tau_e(\lambda_e)\eta_e(u_e)}, \ \left(\tau_e(\lambda_e) > 0,\ \lim_{\lambda_e \to 0} \tau_e(\lambda_e) = 0\right)$$

is the Fourier-Stieltjes transform of some function $\mu_{\lambda_e}^{(\alpha)}(t) \in NBV(-\infty;\infty)$, (i.e. $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\mu_{\lambda_e}^{(\alpha)}(t) = 1$ and $\left(\int_{-\infty}^{-\delta} + \int_{\delta}^{\infty}\right) \left| d\mu_{\lambda_e}^{(\alpha)}(t) \right| \to 0$ for $\lambda_e \to \infty, 1 \le e \le n$).

Then:

I If

$$\left\| f(x) - Q_{\lambda}^{(\alpha)}(f;x) \right\|_{L^p(R_n)} = 0\left(\sum_{e=1}^{n} \tau_e(\lambda_e)\right) \ (4)$$

at the $\lambda \to \infty$, same time $f(x) = 0$ almost everywhere on R_n.

II The following relations are equivalent:

$$\text{(A)}\ \left\| f(x) - Q_{\lambda}^{(\alpha)}(f;x) \right\|_{L^p(R_n)} = O\left(\sum_{e=1}^{n} \tau_e(\lambda_e)\right) \ (5)$$

at $\lambda \to \infty$. (This means that $\lambda_e \to \infty$ for each one $1 \le e \le n$ separately)

(B). There exists a bounded measure ν on R_n and a function $\ell(x) \in L^p(R_n)$ such that for each $\psi(x) \in M_F^e(\psi)$ the relation holds

$$\int_{R_n} r_{\psi}(x) f(x) dx = \begin{cases} \int_{R_n} \psi(x) d\nu(x) & npu \quad p = 1 \\ \int_{R_n} \psi(x)\ell(x) dx & npu \ 1 < p < \infty \end{cases} \ (6)$$

Proof: Let us consider the case $1 < p < \infty$.

I According to (c) we have:

$$\left\| f(x) - Q^{(\alpha)}_{e,\lambda_e}(f;x) \right\|_{L^p(R_n)} = 0(\tau_e(\lambda_e)),\ (1 \le e \le n), \quad (7)$$

for $\lambda_e \to \infty$. Then for any function $\psi(x) \in F$ we will have:

$$\lim_{\lambda_e \to \infty} \int_{R_n} \frac{f(x) - Q^{(\alpha)}_{e,\lambda_e}(f;x)}{\tau_e(\lambda_e)} \psi(x)dx = 0,\ (1 \le e \le n) \quad (8)$$

Since the singular integral (2) is a convolution type integral, we find

$$\int_{R_n} \frac{f(x) - Q^{(\alpha)}_{e,\lambda_e}(f;x)}{\tau_e(\lambda_e)} \psi(x)dx = \int_{R_n} \frac{\psi(x) - Q^{(\alpha)}_{e,\lambda_e}(\psi;x)}{\tau_e(\lambda_e)} f(x)dx \quad (9)$$

for each $\psi(x) \in F$.

Moreover, from $\psi(x) \in M^{\,e}_{F}(\psi)$ the theorem on the convolution of Fourier transforms we have:

$$\left[\frac{\psi(x) - Q^{(\alpha)}_{e,\lambda_e}(f;x)}{\tau_e(\lambda_e)} \right]^{\wedge}(u) = \frac{1 - G^{(\alpha)}_{e,\lambda_e}(u_e)}{\tau_e(\lambda_e)} \psi^{\wedge}(u) =$$

$$= \left[\mu^{(\alpha)}_{\lambda_e}(t_e) \right]^{\vee}(u_e) r^{\wedge}_{\psi}(u) = \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} r_{\psi}(x - t_e) d\mu^{(\alpha)}_{\lambda_e}(t_e) \right]^{\wedge}(u).$$

From here, due to the uniqueness of Fourier transforms, we find:

$$\frac{\psi(x) - Q^{(\alpha)}_{e,\lambda_e}(f;x)}{\tau_e(\lambda_e)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} r_{\psi}(x - t_e) d\mu^{(\alpha)}_{\lambda_e}(t_e)$$

From the last equality it follows that

$$\left\| \frac{\psi(x) - Q^{(\alpha)}_{e,\lambda_e}(f;x)}{\tau_e(\lambda_e)} - r_{\psi}(x) \right\|_{L^p(R_n)} \le$$

$$\le \frac{1}{\sqrt{2\pi}} \left\| r_{\psi}(x - t_e) - r_{\psi}(x) \right\|_{L^p(R_n)} \left| d\mu^{(\alpha)}_{\lambda_e}(t_e) \right| \to 0$$

at $\lambda_e \to \infty,\ (1 \le e \le n)$.

From this we have that

$$\lim_{\lambda_e \to \infty} \int_{R_N} \frac{\psi(x) - Q^{(\alpha)}_{e,\lambda_e}(\psi;x)}{\tau_e(\lambda_e)} f(x)dx = \int_{R_n} r_{\psi}(x) f(x)dx$$

For $f(x) \in L^p(R_n)$.

Therefore, by virtue of (8) and (9)

$$\int_{R_n} r_\psi(x) f(x) dx = 0$$

for $r_\psi(x) \in F$. From this it follows that $f(x) = 0$ almost everywhere on R_n.

II (A) $\Longrightarrow$ (B). Taking into account (C) from (5) it follows

$$\left\| f(x) - Q_{e,\lambda_e}^{(\alpha)}(f;x) \right\|_{L^p(R_n)} = 0(\tau_e(\lambda_e)) \quad (\lambda_e \to \infty, 1 \le e \le n)$$

Then, by the weak compactness theorem (see [4], p. 16), there exists a function $e(x) \in L^p(R_n)$ and subsequences of numbers e_i $(\lim_{e_i \to \infty} \lambda_{e_i} = \infty)$ such that

$$\lim_{e_i \to \infty} \int_{R_n} \frac{f(x) - Q_{e,\lambda_{e_i}}^{(\alpha)}(f;x)}{\tau_e(\lambda_{e_i})} \psi(x) dx = \int_{R_n} \psi(x) e(x) dx \tag{10}$$

for any function $\psi(x) \in F$.

Since the singular integral (2) is a convolution type integral, then taking into account (10) we find:

$$\begin{aligned} \lim_{\lambda_e \to \infty} \int_{R_n} \frac{f(x) - Q_{e,\lambda_e}^{(\alpha)}(f;x)}{\tau_e(\lambda_e)} \psi(x) dx &= \lim_{\lambda_e \to \infty} \int_{R_n} \frac{\psi(x) - Q_{e,\lambda_e}^{(\alpha)}(\psi;x)}{\tau_e(\lambda_e)} f(x) dx = \\ &= \int_{R_n} r_\psi(x) f(x) dx \end{aligned} \tag{11}$$

From the comparison of (10) and (11) we have:

$$\int_{R_n} r_\psi(x) f(x) dx = \int_{R_n} \psi(x) e(x) dx$$

those. true (B).

Now we will prove (B) $\Longrightarrow$ (A). Since

$$\theta_{\lambda_e}^{(\alpha)}(t) = \int_{R_n} \frac{\psi(x) - Q_{e,\lambda_e}^{(\alpha)}(\psi;x)}{\tau_e(\lambda_e)} f(x) dx = \int_{R_n} \frac{f(x) - Q_{e,\lambda_e}^{(\alpha)}(f;x)}{\tau_e(\lambda_e)} \psi(x) dx = \theta_{\lambda_e}^{(\alpha)}(\psi)$$

Then, as in the proof of relation (b) $\Longrightarrow$ (c) of the theorem, we have:

$$\frac{f(x) - Q_{e,\lambda_e}^{(\alpha)}(f;x)}{\tau_e(\lambda_e)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e(x + t_e) d\mu_{\lambda_e}^{(\alpha)}(t_e), \ (1 \le e \le n)$$

or

$$\left\| \frac{f(x) - Q_{e,\lambda_e}^{(\alpha)}(f;x)}{\tau_e(\lambda_e)} \right\|_{L^p(R_n)} \le \frac{1}{\sqrt{2\pi}} \|e(x)\|_{L^p(R_n)} \int_{-\infty}^{\infty} \left| d\mu_{\lambda_e}^{(\alpha)}(t_e) \right| \le M$$

regardless of $\lambda_e, (1 \le e \le n)$, i.e.

$$\left\| f(x) - \theta_{e,\lambda_e}^{(\alpha)}(f;x) \right\|_{L^p(R_n)} = O(\tau_e(\lambda_e)), \, (\lambda_e \to \infty, 1 \le e \le n)$$

Taking into account (d) from the last equality, we find:

$$\left\| f(x) - \theta_{\lambda_e}^{(\alpha)}(f;x) \right\|_{L^p(R_n)} = O\left(\sum_{e=1}^{n} \tau_e(\lambda_e) \right)$$

at $\lambda \to \infty$, i.e. true (A).

The theorem is proved for $1 < p < \infty$ and for $p = 1$ the theorem is proved similarly.

Let us apply the theorems to a specific Fejér linear operator, i.e. the Fejér singular integral:

$$\sigma_\lambda(f;x) = \frac{1}{\prod_{e=1}^{n} (2\pi\lambda_e)} \int_{R_n} f(x-t) \prod_{e=1}^{n} \left(\frac{\sin \frac{1}{2}\lambda_e t_e}{\frac{1}{2} t_e} \right)^2 dt \quad (12)$$

in case $1 \le p \le 2$. In this case $\alpha = 1$ and

$$K_\lambda(t) = \frac{1}{\prod_{e=1}^{n} (2\pi\lambda_e)} \prod_{e=1}^{n} \left(\frac{\sin \frac{1}{2}\lambda_e t_e}{\frac{1}{2} t_e} \right)^2 = \prod_{e=1}^{n} K_{e,\lambda_e}(t_e).$$

Because

$$\left[K_{e,\lambda_e}(t_e)\right]^\wedge(u_e) = \begin{cases} \frac{1}{\sqrt{2\pi}} \left(1 - \frac{|u_e|}{\lambda_e} \right) npu \, |u_e| < \lambda_e \\ 0 \qquad npu \, |u_e| \ge \lambda_e \end{cases}$$

That

$$G_{e,\lambda_e}^{(\alpha)}(u_e) = \begin{cases} 1 - \frac{|u_e|}{\lambda_e} & npu \, |u_e| < \lambda_e \\ 0 & npu \, |u_e| \ge \lambda_e \end{cases}$$

Therefore, for the function $\tau_e(\lambda_e) = \frac{1}{\lambda_e}$ and $u_e(u_e) = |u_e|$ the satisfying relation:

$$\frac{1-G^{(\alpha)}_{e,\lambda_e}(u_e)}{\tau_e(\lambda_e)|u_e|}=\begin{cases}1 \quad npu \quad |u_e|<\lambda_e\\ \dfrac{\lambda_e}{u_e} \; npu \quad |u_e|\geq\lambda_e\end{cases} \quad (1\leq e\leq n) \text{ (13)}$$

It is known [7]that

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\sqrt{\frac{2}{\pi}}\left(\frac{\sin t_e}{t_e}-c_i t_e\right)e^{-t_e\frac{u_e}{\lambda_e}}dt_e=\begin{cases}1 \quad npu \; |u_e|<\lambda_e\\ \dfrac{\lambda_e}{u_e} \; npu|u_e|<\lambda_e,\end{cases} \text{ (14)}$$

Where

$$\overline{C}_i t_e=-\int_{t_e}^{\infty}\frac{\cos u}{u}du,(1\leq e\leq n)$$

Let's introduce the function

$$e_{\lambda_e}(t_e)=\lambda_e\sqrt{\frac{2}{\pi}}\left(\frac{\sin t_e\lambda_e}{t_e}-\overline{C}_i(t_e\lambda_e)\right).$$

Because

$$\int_{-\infty}^{\infty}|e_{\lambda_e}(t_e)|dt_e=\lambda_e\sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty}\left|\frac{\sin t_e\lambda_e}{t_e}-Ci(t_e\lambda_e)\right|dt_e=$$
$$=\sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty}\left|\frac{\sin t_e}{t_e}-\overline{C}_i t_e\right|dt_e\leq M_1<\infty,$$

i.e. $e_{\lambda_e}(t_e)\in L(R_n)$. On the other hand, if through

$$\mu_{\lambda_e}(t_e)=\int_{-\infty}^{t_e}e_{\lambda_e}(u_e)du_e,\ (1\leq e\leq n),$$

let us denote a uniformly bounded measure on R_1, then by virtue of

$$\int_{-\infty}^{\infty}\frac{\sin t_e}{t_e}dt_e=\pi \text{ And } \int_{-\infty}^{\infty}\overline{C}_i t_e dt_e=0,$$

we obtain that $\mu_{\lambda_e}(-\infty)=0$, $\mu_{\lambda_e}(+\infty)=\sqrt{2\pi}$ for all values of λ_e and

$$[\operatorname{var}\mu_{\lambda_e}(t_e)]_{-\infty}^{\infty}=\int_{-\infty}^{\infty}|e_{\lambda_e}(u_e)|du_e\leq M_2<\infty$$

Comparison of (13) and (14) shows that the function

$$\frac{1-C^{(\alpha)}_{e,\lambda_e}(u_e)}{\tau_e(\lambda_e)\eta_e(u_e)}$$

is the Fourier-Stieltjes transform of a normalized function with bounded variation.

Consequently, the conditions of the theorem are satisfied for the Fejér singular integral. Therefore, we have.

I. Let $f(x)\in L^p(R_n),\ (1\le p\le 2)$. Then in order for the relation to take place

$$\left\|\sigma_\lambda(f;x)-f(x)\right\|_{L^p}=0\left(\sum_{e=1}^{n}\frac{1}{\lambda_e}\right),$$

for $\lambda_e\to\infty$, $(1\le e\le n)$ it is necessary and sufficient that $f(x)=0$ almost everywhere on R_n.

II. Let $f(x)\in L^p(R_n),\ (1\le p\le 2)$. In order for the relation to take place

$$\left\|\sigma_\lambda(f;x)-f(x)\right\|_{L^p(R_n)}=0\left(\sum_{e=1}^{n}\frac{1}{\lambda_e}\right)$$

for $\lambda_e\to\infty$, $(1\le e\le n)$ it is necessary and sufficient that $f(x)\in \boldsymbol{n}_p(f)$, were

$$\boldsymbol{n}_p(f)=\begin{cases} f(x)\in L(R_n)\big|\, f(x)\in BV(R_n)\ \ npu\ \ p=1 \\ f(x)\in L^p(R_n)\big|\, f(x)\in AC_{loc}(R_1)\ \ no\ x_e \\ u\ \dfrac{\partial f(x)}{\partial x_e}\in L^p(R_n),\ (1\le p\le 2,\ 1\le e\le n). \end{cases}$$

2. Conclusion

In this paper, the order and class of saturation were determined. α -singular integral of general form in the metric of space $L^p(R_n)$. The results obtained were applied to determine the order and saturation class of a specific Fejér singular integral

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