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A trapezoidal approach to cost optimization in M/M/1 queueing systems

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Abstract

The study addresses the problem of cost optimization in an M/M/1 queueing system by incorporating fuzzy logic to manage uncertainties in system parameters. By modeling imprecise inputs such as arrival rate, service rate and cost factors as fuzzy numbers, the research aims to develop a more realistic and flexible approach to minimizing the total operating cost of the system. In order to deliver more realistic findings for the model under discussion than crisp results, we want to create the total cost function and further compute the total optimal cost of the system in a fuzzy environment. Lastly, the model's numerical and graphical analysis has been done for trapezoidal numbers.

Keywords: M/M/1 Queue; Fuzzified optimization; Optimal total cost; Fuzzified Optimal total cost; Crisp model; Fuzzy model

1. Introduction

Fuzzy set theory and logic may be used to manage and describe fuzzy information that implies imprecise and ambiguous information for decision making Belloman and Zadeh (1970). According to Mishra and Yadav (2010), Mishra and Shukla (2009), Priya and Sudhesh (2018), Sharma (2016), Singh et al. (2016), Sundari and Palaniammal (2015), Singh B B et al. (2020), Palaniammal and Pradeep (2025) Markovian queues have been a major focus in queuing theory research. Fuzzy queuing models have been found to be more effective than crisp ones in Markovian queues because they are more realistic in real-world scenarios. For instance, it is more possibilistic than probabilistic to discuss the mean arrival rate, mean service rate or both. However, we also concur that whereas arrivals and services at service stations are entirely probabilistic and their numerical formulations are possibilistic.

In addition, fuzzy queuing models have a wider variety of applications than crisp ones since they are more realistic (Vide, Prado, and Fuente, 2010). Queuing models may be divided into two categories. The first is referred to as descriptive and the second as normative. Normative queuing models are those that should be the best fit for the circumstances, whereas descriptive models are ones that actually occur in real-world settings. This model optimizes several queuing model characteristics, including arrival, service, number of servers, queue discipline and controls. Consequently, the descriptive model is a real-situation queuing model, whereas the normative model is an aspirational one. Queuing decision models, sometimes referred to as design and control models are the second category of queuing models.

This class of models makes an effort to determine the model's optimal parameters. Among control models, queuing models' service control is dependent on metrics like service rate, server count, queue discipline, or a mix of these. Changing the number of consumers coming, allocating arrivals to certain servers, or establishing tolls or other workable restrictions, such as structuring the physical area and working shift can also help manage arrivals.

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Recently, there has been a shift in the tendency toward optimizing the queuing model with unknown input data. The fuzzy paradigm is used to try to determine the model based on the input data's uncertainty. Several approaches have been developed thus far to solve design and control models of queues in the form of performance measures where cost coefficients, arrival, and service characteristics are known with precision.

According to Prameela and Kumar (2019) and Palpandi and Geetharamani (2013), Shanmugasundaram et al. (2015), fuzzy optimization techniques are used in the fuzzy paradigm. The goal of the intervention in this case is to evaluate the influence on the system's typical functioning. According to Barak and Fallahnezhad (2012), Chen et al. (2020), Fathi-Vajargah and Ghasemalipour (2016), Enrique and Enrique (2014), Kannadasan and Sathiyamoorthi (2018), Gou et al. (2017), and Hidayah et al. (2019), Singh B B et al. (2020), Palaniammal and Pradeep (2024) fuzzy queuing decision models that can thoroughly examine and explore such models are required in order to respond to such a situation.

Both types of models are attempted to be discussed in the study. Because they are more realistic and useful than their traditional counterparts. Queuing models in fuzzy environments are designed and controlled by optimizing the queuing model in its actual scenario with one or more parameters that are unclear. In this article, we address the fuzzy optimization cost of the Markovian queuing model with a single server. This involves fuzzifying the parameters using an appropriate fuzzy set, evaluating specific fuzzy operations and defuzzifying the parameters using the signed distance method. The optimization and computation process is completed.

Building the total cost function in a fuzzy environment and optimizing is our goal. This mathematical intervention gives us for its optimum performance measure, which is optimal total cost.

2. Notations and Assumptions

The following are the notations and assumptions used in this paper:

2.1. Notations

(TC) = Optimal Total Cost (OTC),
$\_k$ = Service cost per unit (SC),
$\_c$ = Waiting cost per unit (WC),
$\_\lambda$ = Arrival rate of customer (ARC),
$\_\mu$ = Service rate (SR),
$\_\mu^*$ = Optimal Service Rate (OSR),
$\_k^*$ = Fuzzified Service cost per unit (FSC),
$\_c^*$ = Fuzzified Waitingcost per unit (FWC),
$\_\lambda^*$ = Fuzzified Arrival Rate (FAR),
$\_(TC)^*$ = Fuzzified Optimal Total Cost (FOTC)

3. Model Development and Analysis

Here, we examine M/M/1 : FCFS / ∞ / ∞ both arrival and service follow the Poisson probability rule. The estimated number of clients in the line and the system's projected waiting time, as well as the service utilization factor or busy period, are some of the queuing model's operational features and performance metrics. These operation characteristics are the focus of most articles, but there is a lack of study on the queuing system's optimal overall cost, especially in fuzzy environments. Two key expenses—waiting costs and service costs—that are in conflict with one another are the basis for the cost model of a queuing system. This suggests that waiting would become more expensive if the quality of service declined. The ideal total cost of the queuing system under discussion can be used to describe the cost model as a performance metric.

The cost model can be expressed as

$$\_(TC) = \_k\_\mu + \_c * \_E(n^*)$$

$$\_(TC) = \_k\_\mu + \frac{\_c\_\lambda}{\_\mu - \_\lambda}$$

Where $\_k$ = cost of service rate per unit time, $\_c$ = cost of waiting customers per unit time and $\_E(n^*)$= average number of customers in the system.

3.1. Fuzzy mathematical formulation

Total model cost function is defined as $\_(TC) = \_k\_\mu + \_\frac{\_c\_\lambda}{\_\mu - \_\lambda}$

For minimum cost w.r.t to service, we have

$$\frac{d}{d\mu}[\_(TC)] = \frac{d}{d\mu}\left[\_k\_\mu + \frac{\_c\_\lambda}{\_\mu - \_\lambda}\right] = \_k - \frac{\_c\_\lambda}{(\_\mu - \_\lambda)^2} = 0$$

For minimum cost, we must have

$$\frac{d^2}{d\mu^2}[\_(TC)] = \frac{2\_c\_\lambda}{(\_\mu - \_\lambda)^3} > 0$$

Further we define a trapezoidal number $A^* = (a^*, b^*, c^*, d^*)$ with membership function

$$\mu_A(x) = \begin{cases} 0 & x^* \leq a^* \\ \frac{x^* - a^*}{b^* - a^*}, & a^* \leq x^* < b^* \\ 1 & b^* \leq x^* < c^* \\ \frac{d^* - x^*}{d^* - c^*}, & c^* \leq x^* < d^* \\ 0 & x^* > d^* \end{cases}$$

Now we wish to fuzzify cost coeffecients and arrival rates $\_k, \_c, \_\lambda$ with the help of trapezoidal fuzzy numbers as $\_k^*, \_c^*, \_\lambda^*$ respectively.

$$\_k^* = (\_k_1, \_k_2, \_k_3, \_k_4)$$

$$\_c^* = (\_c_1, \_c_2, \_c_3, \_c_4),$$

$$\_\lambda^* = (\_\lambda_1, \_\lambda_2, \_\lambda_3, \_\lambda_4)$$

$$\_(TC)^* = \_k^*\_\mu + \frac{\_c^*\_\lambda^*}{\_\mu - \_\lambda^*}$$

Which implies that

$$\_(TC)_* = (\_k_1\_\mu, \_k_2\_\mu, \_k_3\_\mu, \_k_4\_\mu) + \left(\frac{\_c_1\_\lambda_1}{-\lambda_1}, \frac{\_c_2\_\lambda_2}{-\lambda_2}, \frac{\_c_3\_\lambda_3}{-\lambda_3}, \frac{\_c_4\_\lambda_4}{\mu - \lambda_4}\right)$$

$$\_(TC)_* = (P, Q, R, S) where \ P = \_k_1\mu\,\_c_1, Q = \_k_2\mu\,\_c_2, R = \_k_3\mu\,\_c_3, S = \_k_4\mu - \frac{\_c_4\_\lambda_4}{\mu - \lambda_4}$$

Now we define

$$\_(TC)_{*\ ds} = \frac{1}{2}\left[\int_0^1 (\_c_L(\varepsilon) + \_c_R(\varepsilon))\varepsilon \, d\varepsilon\right]$$

$$\frac{d}{d\mu}\_(TC)_{*\ ds} = \frac{1}{4}\left[(\_k_1 + \_k_2 + \_k_3 + \_k_4) - \frac{\_c_4\_\lambda_4}{\mu - \lambda_4}\right]$$

For minimum $\frac{d}{d\mu}\_(TC)^*{}_{ds} = 0$ with sufficient condition

$\frac{d^2}{d\mu^2}\_\_(TC)^*{}_{ds} = \frac{1}{2}\left(\frac{\_c_4\_\lambda_4}{(\mu - \lambda_4)^3}\right) > 0$ which ultimately gives us $(\_k_1 + \_k_2 + \_k_3 + \_k_4) * (\mu - \lambda_4)^2 - \_c_4\_\lambda_4 = 0$

4. Numerical Results and Graphical Analysis

4.1. Results obtained for crisp and fuzzy model

A Python program is used to solve this nonlinear equation and Fuzzy input parameters can be effectively handled and interpreted using signed distance defuzzification method. After Defuzzification via Signed Distance Method we have Computed numerical results and graphical analysis for the optimal service rate _μ and optimal total cost _*(TC)* . We got the Tables 4-6 offer the ideal findings, which are readily compared to Tables 1-3's results for the crisp model.

The results obtained for the crisp model are presented in Tables 1-3.

The results obtained for the fuzzy model are presented in Tables 4-6.

Table 1 Computation table for Service Cost and Optimal Total Cost

	_*k*	_*c*	_*λ*	_*μ*	_*(TC)*
Case 1	23	15	10	12.55	347.47
	25	15	10	12.45	372.47
	27	15	10	12.36	397.28
	29	15	10	12.27	421.91

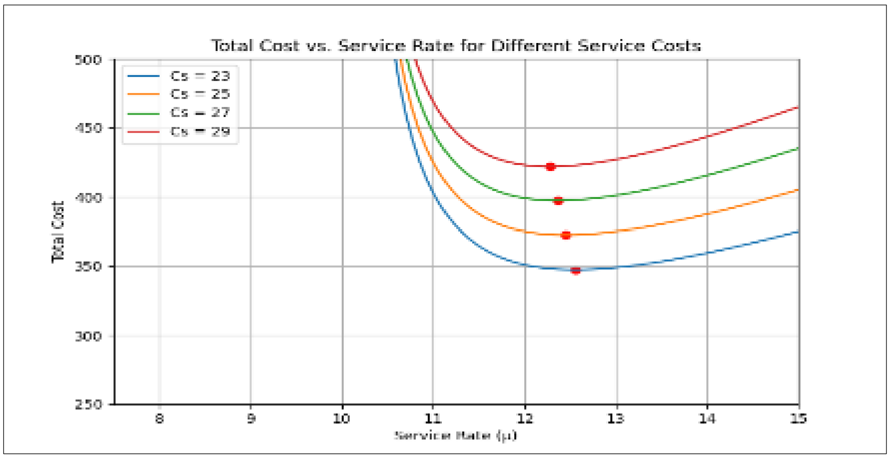

Figure 1 Service cost Vs Total cost

Table 2 Computation table for Waiting Cost and Optimal Total Cost

	_*c*	_*k*	_*λ*	_*μ*	_*(TC)*
Case 2	15	23	10	12.55	347.47
	17	23	10	12.72	355.06
	19	23	10	12.87	362.21
	21	23	10	13.02	368.99

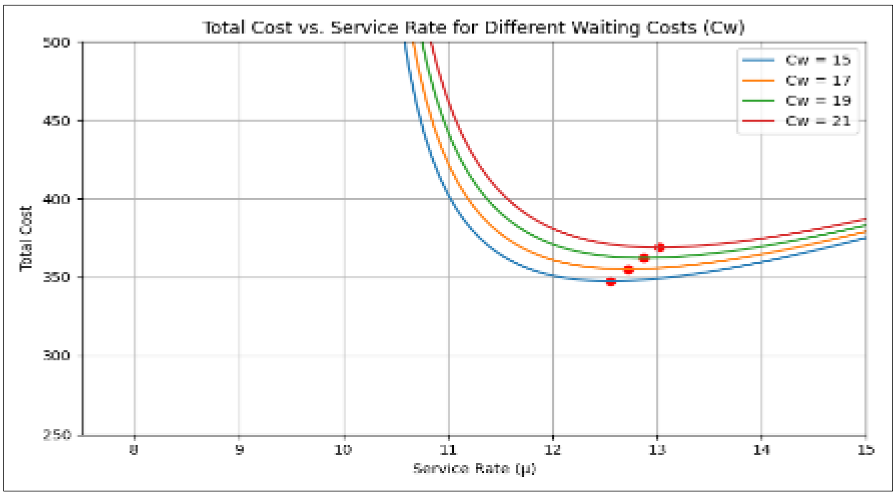

Figure 2 Waiting cost Vs Total cost

Table 3 Computation table for Arrival rate and Optimal Total Cost

	_λ	_c	_k	__μ	_(TC)
Case 3	10	15	23	12.55	347.47
	12	15	23	14.79	404.68
	14	15	23	17.02	460.99
	16	15	23	19.23	516.59

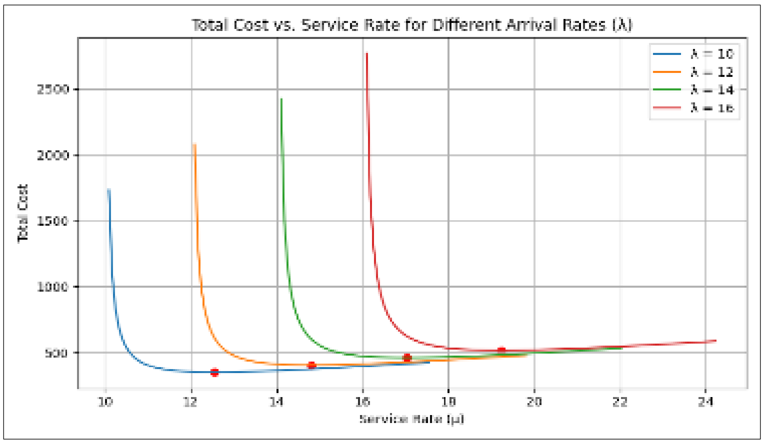

Figure 3 Arrival rate Vs Total cost

4.2. Sensitivity analysis

A sensitivity analysis is a comprehensive examination of the model's variation-propensity.

Table 1 illustrates how the crisp model's overall ideal cost rises in tandem with the service cost per unit. It is noteworthy that the overall optimum cost of the model rises in tandem with increases in waiting costs and waiting times per unit in Table 2. As shown in Table 3 and Fig. 1, the overall optimum cost of the model likewise rises anytime the customer arrival rate to the service channel does.

Table 4 illustrates that the overall optimal fuzzy cost of the fuzzy model under discussion rises in proportion to the increase in fuzzy service cost per unit. Table 5 illustrates that when the fuzzy waiting cost per unit rises, the model's overall optimal fuzzy cost also rises. Finally, Table 6 shows that if the fuzzy arrival rate of consumers to the service channel rises, the model's overall optimal fuzzy cost also rises.

Table 4 Computation table for Fuzzified Service cost _k* & Fuzzified Optimal Total Cost _(TC)*

	_k*				_c*				_λ*				_μ*	_(TC)*
	_k1*	_k2*	_k3*	_k4*	_c1*	_c2*	_c3*	_c4*	_λ1*	_λ2*	_λ3*	_λ4*	_μ*	_(TC)*
Case 1	20	22	24	26	16	18	20	22	12	14	16	18	18.52	506.92
	22	24	26	28	16	18	20	22	12	14	16	18	18.37	543.82
	24	26	28	30	16	18	20	22	12	14	16	18	18.25	580.44
	26	28	30	32	16	18	20	22	12	14	16	18	18.13	616.82

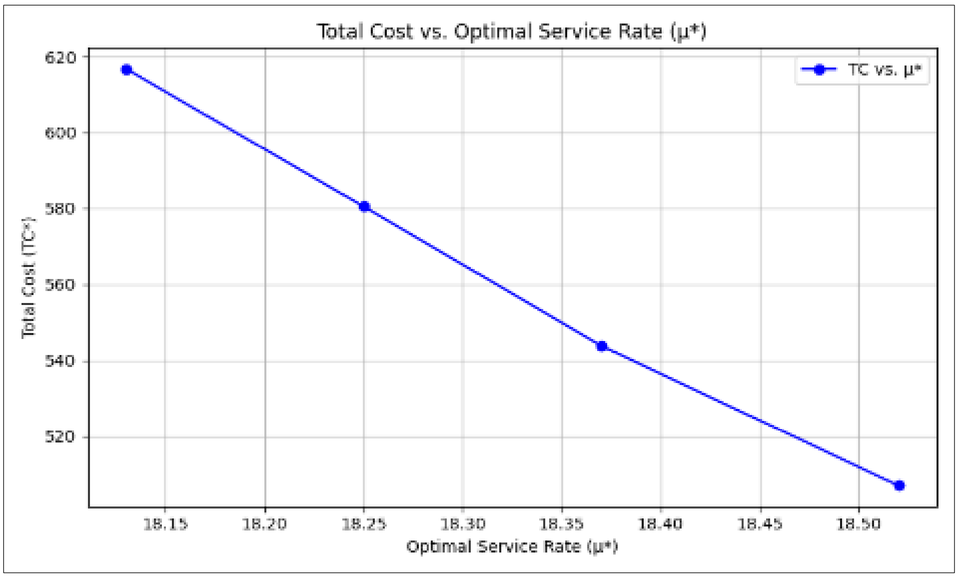

Figure 4 Fuzzified Service cost Vs Fuzzified Optimal Total Cost

Table 5 Computation table for Fuzzified Waiting cost _c* & Fuzzified Optimal Total Cost _(TC)*

	_c*				_k*				_λ*				_μ*	_(TC)*
	_c1*	_c2*	_c3*	_c4*	_k1*	_k2*	_k3*	_k4*	_λ1*	_λ2*	_λ3*	_λ4*	_μ*	_(TC)*
Case 2	16	18	20	22	20	22	24	26	12	14	16	18	18.52	506.92
	18	20	22	24	20	22	24	26	12	14	16	18	18.70	515.23
	20	22	24	26	20	22	24	26	12	14	16	18	18.87	523.15
	22	24	26	28	20	22	24	26	12	14	16	18	19.03	530.74

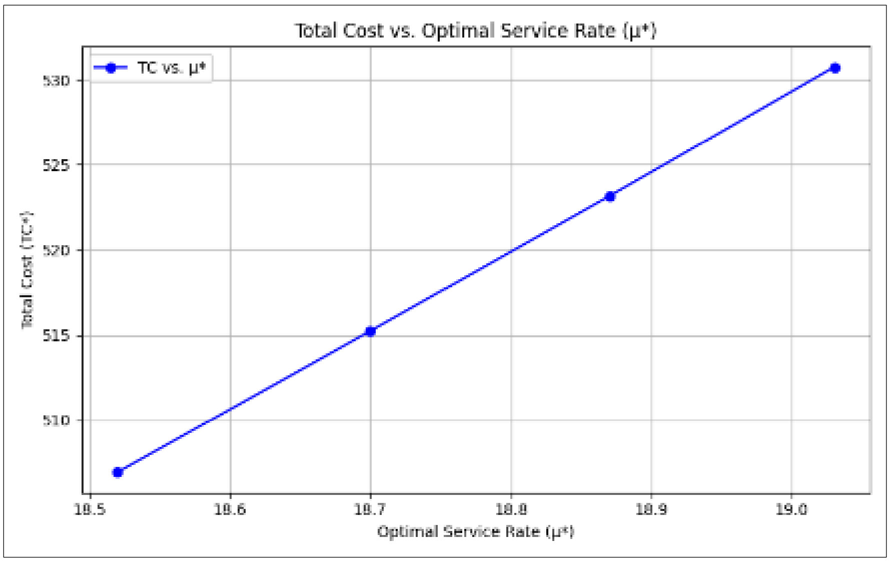

Figure 5 Fuzzified Waiting cost Vs Fuzzified Optimal Total Cost

Table 6 Computation table for Fuzzified Arrival Rate _λ* & Fuzzified Optimal Total Cost _(TC)*

	_λ*				_c*				_k*				_μ*	_(TC)*
	_λ1*	_λ2*	_λ3*	_λ4*	_c1*	_c2*	_c3*	_c4*	_k1*	_k2*	_k3*	_k4*	_μ*	_(TC)*
Case 3	12	14	16	18	16	18	20	22	20	22	24	26	18.52	506.92
	14	16	18	20	16	18	20	22	20	22	24	26	20.74	563.38
	16	18	20	22	16	18	20	22	20	22	24	26	22.96	619.24
	18	20	22	24	16	18	20	22	20	22	24	26	25.16	674.59

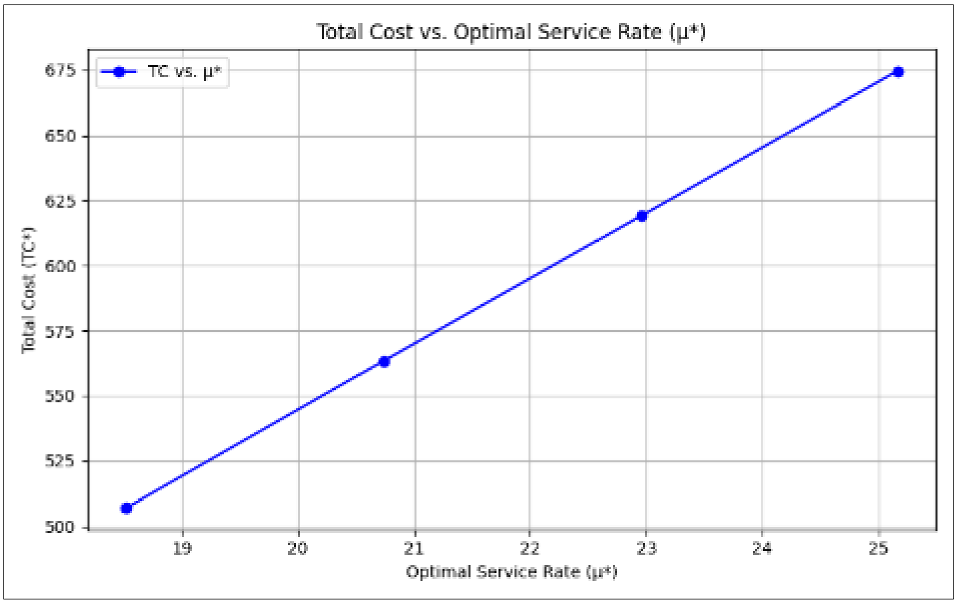

Figure 6 Fuzzified Arrival rate Vs Fuzzified Optimal Total Cost

One essential metric for assessing the model's suitability for use is correlation. As a result, there is a positive association between the overall optimal cost of the model as a variable and the service, waiting and arrival rate as another variable in both the crisp and fuzzy situations. The connection in the former scenario is less positive than in the latter, which is the single fundamental difference between the two contexts.

5. Conclusion

In the current study, we created a fuzzy environment queuing model that is more accurate in situations where we have no control over measurements. An enhanced approach to the current queuing model is the computation of the queuing model's overall optimal cost using a single server in a fuzzy environment. In both the fuzzy and the crisp settings, the outcomes from Section 4 are readily comparable.

Fuzzy cost models have a wide range of real-world applications, especially in systems where costs, times or probabilities are uncertain or imprecise by extending such a Markovian queuing models to a fuzzy environment. The development of more effective waiting line systems to Manufacturing and Production Systems, Healthcare and Hospital Management, Cloud Computing / IT Services, Transportation and Traffic Systems, Retail and Customer Service, Maintenance & Repair Scheduling. Fuzzy systems, neural networks and intuitionistic fuzzy techniques will be more practical for studying such queuing models in the future.

Compliance with ethical standards

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Disclosure of conflict of interest

No conflict of interest to be disclosed.

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