International Journal of Science and Research Archive
eISSN: 2582-8185
Cross Ref DOI: 10.30574/ijsra
Journal homepage: https://ijsra.net/

(RESEARCH ARTICLE)

Check for updates

Enhancing student performance in mathematics through concrete-representational-abstract (CRA) approach

Ellen Grace Pioquinto Bernadez * and Jennifer Medrano Montero

Department of Graduate Studies, North Eastern Mindanao State University (NEMSU), Philippines.

International Journal of Science and Research Archive, 2025, 14(03), 1732-1738

Publication history: Received on 18 February 2025; revised on 29 March 2025; accepted on 31 March 2025

Article DOI: https://doi.org/10.30574/ijsra.2025.14.3.0887

Abstract

This study examines the effectiveness of the Concrete-Representational-Abstract (CRA) approach, grounded in Jerome Bruner's cognitive development theory, in enhancing students' performance among Grade 7 students at St. Christine National High School, Lianga, Surigao del Sur, Philippines. Using a quasi-experimental design, two groups—experimental (CRA) and control (conventional approach)—were assessed via pre- and post-tests on operations of integers, simplifying numerical expressions, and identifying the absolute value of an integer and its meaning. The CRA group progressed through three stages: hands-on manipulatives (concrete), visual representations (representational), and symbolic problem-solving (abstract), while the control group received traditional/conventional approach. Initial pre-test scores were comparable (Control: M = 15.22, SD = 3.97; Experimental: M = 15.75, SD = 5.02), but post-tests revealed a marked divergence: the experimental group achieved a mean score of 24.22 (SD = 6.62), reflecting an 8.47-point improvement, whereas the control group scored 21.28 (SD = 8.73), with a 6.06-point gain. This 39.8% greater improvement in the CRA group underscores its efficacy in fostering deeper conceptual mastery and reducing performance variability, attributed to structured scaffolding that bridges tangible experiences to abstract reasoning. The results demonstrate the effectiveness of the CRA approach in improving Grade 7 learners' mathematics performance, underscoring its value in enhancing mathematics education and addressing educational challenges faced by Filipino learners.

Keywords: Concrete-Representational-Abstract (CRA) Approach; Students' Performance; Filipino

1. Introduction

This study explores the effectiveness of the Concrete, Representational (Pictorial), Abstract model (CRA) in enhancing students' understanding of mathematical concepts. This is grounded in Jerome Bruner's theory of cognitive development, which posits that learning progresses through three stages: enactive (action-based), iconic (image-based), and symbolic (language-based). By leveraging this progression, the CRA approach facilitates learning through a structured sequence, beginning with hands-on experiences using concrete objects, followed by visual representations, and culminating in abstract symbolic notation.

The author emphasizes the impact of the CRA approach in selected topics in mathematics for grade 7 learners. This method not only aids students in grasping mathematical concepts but also addresses significant learning gaps. Research indicates that hands-on experiences and systematic interventions are crucial for improving students' understanding and retention of mathematical concepts, particularly for those facing learning difficulties. By employing concrete materials initially, students can build a solid foundation before transitioning to more abstract representations, thus enhancing their overall mathematical competence.

* Corresponding author: Ellen Grace P. Bernadez.

Copyright © 2025 Author(s) retain the copyright of this article. This article is published under the terms of the Creative Commons Attribution Liscense 4.0.

Identified gaps in the study include the insufficient mastery of basic mathematical skills among Filipino students, who are reportedly lagging five to six years behind their peers in learning competencies, according to the Department of Education (DepEd) based on the 2022 PISA results. They scored substantially below the OECD average in mathematics, with only 16% achieving at least Level 2 proficiency, compared to the OECD average of 69%. The research highlights the necessity for targeted interventions and effective teaching strategies that can bridge these educational disparities. Last school year, the Mean Percentage Score (MPS) for grade 7 mathematics at St. Christine National High School was 51.19. The analysis revealed a need for remediation, as only 1 student (0.58%) was proficient, while 91 were low proficient and 79 were nearly proficient. Addressing these issues is crucial to ensure students develop necessary skills in mathematics.

Ultimately, this study contributes valuable insights into effective teaching strategies that can enhance mathematical understanding among students. By focusing on the CRA model, it provides educators with a framework for teaching fundamental operations involving integers, thereby fostering improved conceptual understanding and procedural skills. The evidence supporting the CRA approach's effectiveness in enhancing knowledge retention and addressing learning gaps reinforces its potential impact on creating a more competent and innovative society. As such, this research not only aims to improve individual student outcomes but also seeks to contribute to broader educational reforms in the Philippines.

2. Literature review

Many middle school students who have difficulties with mathematics struggled in their earlier grades. However, some students who had been reasonably adept at mathematics may begin to struggle as they work with increasingly difficult content involving rational numbers, including integers. As students work on middle school mathematics problems, they may rely on rules that no longer apply (Bryant, D. P,et.al, 2020).

This struggle with mathematical concepts contributes to broader issues in student performance. The poor performance of students in mathematics has become a concern for many. Despite numerous solutions proposed by experts, the issue persists. Many believe that a lack of foundational knowledge in basic mathematics is a primary cause of this problem. Integer operations are fundamental in mathematics, and proficiency in these operations is essential before advancing to more complex topics. However, students often struggle with learning integer operations (Zainudin et al., 2022). Consequently, finding effective teaching methods is crucial.

However, this study aims to provide a strategy to enhance student's performance in Mathematics. And the author uses integer chips as manipulative during the concrete stage. Manipulatives are real-world materials or objects that students can physically interact with to develop mathematical concepts. Incorporating these into mathematics instruction can spark and maintain students' interest, foster active participation, and enhance learning outcomes. Algebra tiles are a particularly effective type of manipulative for teaching algebra, offering a visual representation of operations involving variables and numbers (Abdul-Karim, H., et.al., 2023).

Moreover, Mathematical ideas are abstract mental constructs. To help students grasp these ideas, they must be represented in a more concrete way using external representations. These external representations take the place of the abstract, mental concepts, and they embody the key properties of the concepts (Gallo-Toong, N., 2020). To assist students in acquiring a conceptual understanding of Mathematics, several strategies have been developed. One such strategy is the Concrete-Representational-Abstract method of instruction.

Concrete-Representational-Abstract (CRA) framework addresses multiple learning styles and utilizes multisensory input to get students a true understanding of math concepts. It also supports the development of mathematical skills in students of all learning types at all levels (Ali, S., 2022). During each phase the instructor first models the skill using the phase-specific manipulatives (i.e., blocks, tallies, or numbers), then guides students through one or two problems, and finally asks them to solve a few problems on their own. Once students reach a certain level of mastery on the independent practice problems, they move to the next phase (Ebner, S., etal.,2025).

CRA has its roots in the work of Bruner and Kenney (1965), who defined learning through "Stages of Representation:" Enactive, or learning through movement and action, Iconic, or learning through pictures, and Symbolic, or learning through abstract symbols. The CRA instructional sequence/ approach provides a graduated, conceptually supported line of work to create meaningful connections among concrete, representational, and abstract levels of understanding.

3. Methodology

This research employed a quasi-experimental, pretest-posttest control group design to evaluate the efficacy of the Concrete-Representational-Abstract (CRA) approach in enhancing the mathematics performance of Grade 7 learners. The study aimed to address identified learning gaps in key mathematical concepts, including integer operations, numerical expression simplification, and absolute value, as highlighted by PISA 2022 results and low Mean Percentage Scores (MPS).

The participant pool consisted of two intact Grade 7 sections from St. Christine National High School, each comprising 32 students. One section was designated as the experimental group, which received instruction utilizing the CRA approach, while the other served as the control group, receiving instruction through conventional teaching methods. The researchers developed and utilized a suite of materials, including lesson exemplars, pretest-posttest instruments, and quiz activities. These instruments underwent rigorous validation by a mathematics specialist and a Master Teacher to ensure validity and reliability, followed by pilot testing with a separate Grade 7 section to verify clarity and appropriateness.

Prior to the intervention, formal permissions were obtained from the school principal, District Supervisor, and Division Superintendent. The intervention, spanning four weeks from January 27 to February 21, 2025, involved the experimental group receiving instruction using the CRA approach, which progressively introduced concepts through concrete materials, representational methods, and abstract symbols. Conversely, the control group received instruction using conventional methods, such as lectures. Pretests and posttests were administered to both groups, and quiz activities were employed throughout the intervention.

The collected data were statistically analyzed using descriptive statistics, such as mean and standard deviation, to summarize student performance. Independent samples t-tests were performed to compare the mean scores of the experimental and control groups on the posttest, testing the null hypothesis at a 5% significance level. These analyses will provide insights into the effectiveness of the CRA approach compared to Conventional or traditional instruction in enhancing the mathematics performance of Grade 7 learners.

4. Result and discussion

This section presents the results, analysis and interpretation of data gathered in the study. The sequence of the discussion was based on the objectives and hypothesis of the study.

Table 1 Pretest and Posttest scores of the Learners

Group	**Pretest**		**Post test**	
	Mean	Sd	Mean	Sd
Control	15.219	3.974	21.28	8.73
Experimental	15.75	5.016	24.22	6.62

The analysis reveals a noteworthy difference in performance between the control and experimental groups. While both groups started with similar pretest scores (Control: M=15.22, SD=3.97; Experimental: M=15.75, SD=5.02), the experimental group demonstrated a more substantial improvement in the posttest (M=24.22, SD=6.62) compared to the control group (M=21.28, SD=8.73). The experimental group not only gained 8.47 points, exceeding the control group's gain of 6.06 points by 39.8%, but also exhibited more consistent outcomes, as indicated by a decrease in standard deviation. A concerning trend, however, is the substantial increase in variability within the control group's posttest scores, warranting further investigation. While these descriptive statistics point towards a positive effect of the experimental intervention, further analysis, including significance testing and normality checks, is recommended for definitive conclusions.

Table 2 Test of Normality

	groups	**Kolmogorov-Smirnovb**			**Shapiro-Wilk**		
		Statistics	df	Sig.	Statistics	df	Sig.
scores	Control	.153	32	.054	.944	32	.094
	experimental	.129	32	.190	.959	32	.259

The Kolmogorov-Smirnov and Shapiro-Wilk tests were conducted to assess the normality of data from a control group and an experimental group, both using a significance level (α) of 0.05. The null hypothesis for these tests is that the data are sampled from a population with a normal distribution. Since all p-values (Control group: p=0.054 and Experimental group: p=0.190 for Kolmogorov-Smirnov; Control group: p=0.094 and Experimental group: p=0.257 for Shapiro-Wilk) are greater than 0.05, we fail to reject the null hypothesis for both groups in both tests, indicating no significant evidence to conclude that the data deviate from normality. Given these results, the scores for both the control and experimental groups can be considered normally distributed, making parametric statistical methods, which assume normality, appropriate for further analysis of these data. The Shapiro-Wilk test is generally considered more reliable for smaller sample sizes.

Table 3 Significant difference in the learner's pretest and posttest between two groups

Sources of Variation	**Computed t**	**P-value**	**Decision**	**Conclusion**
Pretest	0.49	0.626	Failed to reject null hypothesis	Not Significant
Posttest	2.06	0.048	reject null hypothesis	Significant

The pretest analysis comparing the control group (mean = 15.22, SD = 3.97) and experimental group (mean = 15.75, SD = 5.02) revealed no statistically significant difference in baseline performance between the two groups. This conclusion is supported by a t-value of 0.49 and a p-value of 0.626, which exceeds the conventional significance threshold of 0.05. The negligible t-value indicates that the observed difference in pretest means (a mere 0.53-point gap) is likely attributable to random sampling variability rather than a meaningful, systematic advantage for either group.

The similarity in starting scores (15.22 vs. 15.75) further underscores that both groups began the study with comparable levels of knowledge or skill, a critical precondition for valid post-intervention comparisons. While the experimental group exhibited slightly greater variability in pretest scores (SD = 5.02 vs. 3.97), this difference was insufficient to produce a statistically meaningful distinction between the groups. These results confirm the baseline equivalence of the groups, thereby strengthening the validity of subsequent posttest comparisons and reducing concerns about pre-existing disparities influencing the study's outcomes. In practical terms, this alignment ensures that any performance differences observed after the intervention can more confidently be attributed to the experimental treatment rather than initial imbalances.

The posttest analysis revealed a statistically significant difference between the experimental and control groups, as evidenced by a t-value of 2.06 and a p-value of 0.048. Because this p-value falls below the conventional significance threshold of 0.05, this result allows us to reject the null hypothesis, indicating that there is a significant difference in student scores between the two groups. Specifically, the experimental group, which received the Concrete-Representational-Abstract (CRA) intervention, achieved a substantially higher posttest mean score of 24.22 compared to the control group's 21.28, reflecting a 2.94-point advantage. This difference, combined with the experimental group's lower posttest standard deviation of 6.62 compared to the control group's 8.73, implies more consistent outcomes among participants who received the CRA approach. Therefore, the data strongly supports the conclusion that the CRA intervention had a meaningful and positive impact on student learning.

These findings collectively suggest that CRA Approach is not only improved performance but also reduced variability in results, highlighting its potential as an effective strategy.

5. Conclusions

Summary

This study investigated the impact of an instructional intervention, specifically the Concrete-Representational-Abstract (CRA) approach, on the academic performance of Grade 7 mathematics learners. The research was conducted at St. Christine National High School in Lianga, Surigao del Sur. The analysis centered on a comparative examination of pre-test and post-test scores between control and experimental groups.

Initially, both groups demonstrated statistically similar pre-test scores, establishing baseline equivalence. This allowed for valid comparisons of post-test performance. Following the intervention, the experimental group exhibited a statistically significant and practically meaningful improvement in post-test scores compared to the control group. The experimental group not only achieved a higher mean score but also demonstrated reduced variability in their results, indicating more consistent learning outcomes.

Statistical analyses, including t-tests and normality tests, confirmed these findings. The t-test revealed no significant difference in pre-test scores between the groups, while a significant difference was found in post-test scores. Normality tests indicated that data from both groups were normally distributed, validating the use of parametric statistical methods.

Furthermore, these analyses, when considered alongside the experimental group's significant improvement and reduced variability, underscore the potential of the CRA approach. However, the unexpected increase in variability within the control group's post-test scores warrants further investigation.

Conclusions

Based on the results of the study, the initial analysis revealed that both groups exhibited statistically similar pre-test scores, establishing a crucial baseline equivalence. The experimental group demonstrated significant post-test improvement, with higher mean scores and reduced variability, validated by t-tests and normality tests. This establishes the CRA approach effectiveness in enhancing student performance in Mathematics. Therefore, the null hypothesis is rejected, indicating a significant difference in student scores between the two groups.

These findings collectively demonstrate the effectiveness of the CRA approach in improving Grade 7 learners' academic performance in mathematics. Specifically, the study indicates that the manipulatives employed during the concrete stage, such as algebra tiles and integer chips, are appropriate and effective. Furthermore, the representational tools, including diagrams and number lines, successfully prepared learners for the abstract stage, where they solved mathematical problems and equations using symbolic notation. In essence, this study provides valuable insights into effective teaching methodologies, highlighting the importance of structured progression from concrete to abstract learning in mathematics education.

Recommendations

In light of the study's findings, the following recommendations are presented.

First, given the demonstrated effectiveness of the Concrete-Representational-Abstract (CRA) approach in enhancing Grade 7 learners' academic performance, educators may integrate this methodology into their instructional practices, particularly in mathematics. Specifically, algebra tiles or integer chips are recommended as concrete manipulatives within the CRA approach for teaching topics such as operations with integers, simplifying numerical expressions, and understanding absolute value.

Educators should also explore and utilize a variety of other concrete materials during the concrete stage, ensuring alignment with the CRA framework's principles.

Furthermore, given the approach's success, educators are encouraged to extend the application of the CRA methodology to other topics within mathematics and across various subject areas, where appropriate.

To facilitate effective implementation, the development and distribution of CRA-based instructional resources, including detailed lesson plans and activity guides, are essential. Educational institutions may actively promote data-driven decision-making, utilizing assessment data to inform instruction and evaluate intervention effectiveness.

Finally, to ensure successful adoption of the CRA approach, providing teachers with adequate professional development and training on its implementation and the utilization of concrete manipulatives is essential.

Compliance with ethical standards

Acknowledgments

The researchers wish to express sincere appreciation to everyone who assisted in completing this study. Their contributions were invaluable, as without them, this work could not have been accomplished. Most importantly, they offer profound thanks to Almighty God, the Creator of all, for the blessings, the gift of life, and the wisdom and knowledge that facilitated the production of this paper.

Disclosure of conflict of interest

No conflict of interest to be disclosed.

References

[1] A, Azmidar., Hartina, Husan. (2022). Enhancing students' mathematical representation ability through mathematics learning. Hipotenusa, 5(2):35-49. doi: 10.36269/hjrme.v5i2.969

[2] A, Azmidar., P, N, Malasari. (2022). Using the Concrete-Representational-Abstract Approach to Enhance Students' Interest in Mathematics Refers to the Primer Mathematical Skills. Jurnal Pendidikan MIPA, 23(3):894-903. doi: 10.23960/jpmipa/v23i3.pp894-903

[3] Abdul-Karim, H., Kasimu, O., Rahaman, A. A., Kanimam, Y. S., Imoro, M., & Dokurugu,M. E. (2023). Assessing the Impact of Algebra Tiles as Visual and Manipulative Aids on Students' Algebraic Understanding. American Journal of Educational Research, 11(10), 705-711.

[4] Ali, Staff. (2022). All About The Concrete Representational Abstract (CRA) Method.https://blog.acceleratelearning.com/concrete-representational-and abstract#:~:text=The%20acronym%20CRA%20stands%20for,from%20using%20the%20CRA%20framework.

[5] Accelerate Learning. (2022). All About the Concrete Representational Abstract (CRA) Method. https://blog.acceleratelearning.com/concrete-representational-and-abstract

[6] AL-salahat, M. M. S. (2022). The effect of using concrete-representational-abstract sequence in teaching the perimeter of geometric shapes for students with learning disabilities. International Journal of Education in Mathematics, Science, and Technology (IJEMST), 10(2), 477-493. https://doi.org/10.46328/ijemst.2403

[7] Bruner, J. S. (1966). Toward a theory of instruction. Harvard University Press.

[8] Bryant, D. P., Bryant, B. R., Dougherty, B., Roberts, G., Pfannenstiel, K. H., & Lee, J. (2020). Mathematics performance on integers of students with mathematics difficulties. The Journal of Mathematical Behavior, 58, 100776.

[9] Bulelwa, Penelope, Stemele., Zaheera, Jina, Asvat. (2024). Exploring Learner Errors and Misconceptions in Algebraic Expressions with Grade 9 Learners Through the use of Algebra Tiles. African Journal of Research in Mathematics, Science and Technology Education, doi: 10.1080/18117295.2024.2334989

[10] Camañan, C. T. (2019). Employing concrete-representation-abstract approach in enhancing mathematics performance. SLONGAN, 4(1), 25-25.

[11] Çetin, H. (2019). Explaining the concept and operations of integer in primary school mathematics teaching: Opposite model sample. Universal Journal of Educational Research, 7, 2, 365-370.

[12] Ebner, S., MacDonald, M. K., Grekov, P., & Aspiranti, K. B. (2025). A Meta-Analytic Review of the Concrete-Representational-Abstract Math Approach. Learning Disabilities Research & Practice, 40(1), 31-42. https://doi.org/10.1177/09388982241292299

[13] Flores, M. M., & Hinton, V. M. (2022). Use of the Concrete–Representational–Abstract Instructional Sequence to Improve Mathematical Outcomes for Elementary Students With EBD. Beyond Behavior, 31(1), 16-28. https://doi.org/10.1177/10742956211072421

[14] Gallo-Toong, N. (2020). The extent of use of concrete-representational-abstract (CRA) model in mathematics. International Journal for Research in Mathematics and Statistics, 6(5), 1-25.

[15] Gallo-Toong, N. (2020). The extent of use of concrete-representational-abstract (CRA) model in mathematics. International Journal for Research in Mathematics and Statistics, 6(5), 1-25.

[16] Harun, Nurul Ain J., et al. (2023). "Assessing Students' Mastery and Misconceptions in the Fundamental Operations on Integers." International Journal of Science, Technology, Engineering and Mathematics, vol. 3, no. 3, pp. 36-55, doi:10.53378/353000.

[17] José, Antonio, Núñez-López., David, Molina-García., José, Luis, González-Fernández., Iván, Fernández-Suárez. (2024). Enhancing the acquisition of basic algebraic principles using algebra tiles. Eurasia journal of mathematics, science and technology education, 20(7):em2473-em2473. doi: 10.29333/ejmste/14750

[18] Larsson, S., & Palm, M. (2023). Successful teaching of multiplication: for knowledge development in mathematics.

[19] Mathematics Hub. "Concrete, Representational, Abstract (CRA)." Mathematics Hub, www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/concrete-representational-abstract-cra/. Accessed 13 Nov. 2024.

[20] Miller, D. J., & Hudson, P. B. (2022). Mathematics learning from concrete to abstract (1968-2021). ERIC. Retrieved from https://files.eric.ed.gov/fulltext/EJ1338995.pdf

[21] Morata, Darlene Eloisa, Estores, Riza C., Famentera, April Rose, et.al.(2024). The use concrete-representational-abstract approach (cra) in improving the adding skills in kindergarten. Retrieved from, https://www.irjmets.com/uploadedfiles/paper//issue_10_october_2024/62957/final/fin_irjmets1729948613.pdf. Accessed 13 Nov. 2024.

[22] Nugroho, S. A., & Jailani, J. (2019). The effectiveness of concrete representational abstract approach (CRA) approach and problem solving approach on mathematical representation ability at elementary school. KnE Social Sciences, 27-36

[23] Peltier C., Morin K. L., Bouck E. C., Lingo M. E., Pulos J. M., Scheffler F. A., Suk A., Mathews L. A., Sinclair T. E., Deardorff M. E. (2020). A meta-analysis of single-case research using mathematics manipulatives with students at risk or identified with a disability. The Journal of Special Education, 54(1), 3–15. https://doi.org/10.1177/0022466919844516

[24] Prosser, S. K., & Bismarck, S. F. (2023). Concrete–Representational–Abstract (CRA) Instructional Approach in an Algebra I Inclusion Class: Knowledge Retention Versus Students' Perception. Education Sciences, 13(10), 1061. https://doi.org/10.3390/educsci13101061

[25] Reyes, J. D. (2021). Effectiveness of Concrete-Representational-Abstract sequence of instruction in probability on select-groups of eight graders. International Journal of Multidisciplinary Research and Growth Evaluation, 2(1), 438-443.

[26] Rogers, J., & Révész, A. (2019). Experimental and quasi-experimental designs (pp. 133–143). Routledge. https://doi.org/10.4324/9780367824471-12

[27] Rosyidah, A. N., Maulyda, M. A., Jiwandono, I. S., Oktaviyanti, I., & Gunawan, G. (2021). Misconceptions and errorss integer operations: A study in preservice elementary school teachers (PGSD). Journal of Physics: Conference Series. doi:10.1088/1742- 6596/1779/1/012078

[28] Satria Adi Nugroho and Jailani, (2019), "The Effectiveness of Concrete Representational Abstract Approach (CRA) Approach and Problem Solving Approach on Mathematical Representation Ability at Elementary School" in International Conference on Meaningful Education, KnE Social Sciences, pages 27–36. DOI 10.18502/kss.v3i17.4620

[29] Servallos, Neil Jayson. (2023). Philippine Star. Retrieved from, https://www.philstar.com/headlines/2023/12/07/2317044/pisa-philippines-5-6-years-behind

[30] Shen, Z. (2023). The effectiveness of concrete-representational-abstract integration framework within word-problem solving for students experiencing mathematics difficulty (Doctoral dissertation).

[31] Witzel, B. S. (2005). The effects of using a concrete-representational-abstract instructional approach on student performance in algebra. Learning Disabilities Research & Practice, 20(3), 172-179.

[32] Zainudin, M., Amin, A. K., & Fatah, D. (2022). Understanding Elementary School Students' Errors in Completing Number Operations. KnE Social Sciences. https://doi.org/10.18502/kss.v7i19.12478